Lesson 5: Negative Exponents and the Laws of Exponents

Classwork

Definition: For any positive number x and for any positive integer n, we define $x^{-n} = \frac{1}{x^n}$.

Note that this definition of negative exponents says x^{-1} is just the reciprocal $\frac{1}{x}$ of x.

As a consequence of the definition, for a positive x and all *integers* b, we get

$$x^{-b} = \frac{1}{x^b}$$

Exercise 1

Verify the general statement $x^{-b} = \frac{1}{x^b}$ for $x = 3$ and $b = -5$.

Exercise 2

What is the value of (3×10^{-2})?

Exercise 3

What is the value of (3×10^{-5})?

Exercise 4

Write the complete expanded form of the decimal 4.728 in exponential notation.

For Exercises 5–10, write an equivalent expression, in exponential notation, to the one given and simplify as much as possible.

Exercise 5

$5^{-3} =$

Exercise 6

$\frac{1}{8^9} =$

Exercise 7

$3 \cdot 2^{-4} =$

Exercise 8

Let x be a nonzero number.

$x^{-3} =$

Exercise 9

Let x be a nonzero number.

$\frac{1}{x^9} =$

Exercise 10

Let x, y be two nonzero numbers.

$xy^{-4} =$

We accept that for positive numbers x, y and all integers a and b,

$$x^a \cdot x^b = x^{a+b}$$

$$\left(x^b\right)^a = x^{ab}$$

$$(xy)^a = x^a y^a$$

We claim:

$$\frac{x^a}{x^b} = x^{a-b}$$ for all *integers* a, b

$$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$$ for any *integer* a

Exercise 11

$$\frac{19^2}{19^5} =$$

Exercise 12

$$\frac{17^{16}}{17^{-3}} =$$

Exercise 13

If we let $b = -1$ in (11), a be any integer, and y be any positive number, what do we get?

Exercise 14

Show directly that $\left(\frac{7}{5}\right)^{-4} = \frac{7^{-4}}{5^{-4}}$.

Problem Set

1. Compute: $3^3 \times 3^2 \times 3^1 \times 3^0 \times 3^{-1} \times 3^{-2} =$
   Compute: $5^2 \times 5^{10} \times 5^8 \times 5^0 \times 5^{-10} \times 5^{-8} =$
   Compute. For a nonzero number, a: $a^m \times a^n \times a^l \times a^{-n} \times a^{-m} \times a^{-l} \times a^0 =$

2. Without using (10), show directly that $(17.6^{-1})^8 = 17.6^{-8}$.

3. Without using (10), show (prove) that for any whole number n and any positive number y, $(y^{-1})^n = y^{-n}$.

4. Show directly without using (13) that $\frac{2.8^{-5}}{2.8^7} = 2.8^{-12}$.