Lesson 3: Numbers in Exponential Form Raised to a Power

Classwork

For any number x and any positive integers m and n,

$$(x^m)^n = x^{mn}$$

because

$$\begin{aligned}(x^m)^n &= (\underbrace{x \cdot x \cdots x}_{m\ times})^n \\ &= \underbrace{(x \cdot x \cdots x)}_{m\ times} \times \cdots \times \underbrace{(x \cdot x \cdots x)}_{m\ times} \qquad (n\ times) \\ &= x^{mn}\end{aligned}$$

Exercise 1

$(15^3)^9 =$

Exercise 2

$((-2)^5)^8 =$

Exercise 3

$(3.4^{17})^4 =$

Exercise 4

Let s be a number.

$(s^{17})^4 =$

Exercise 5

Sarah wrote that $(3^5)^7 = 3^{12}$. Correct her mistake. Write an exponential expression using a base of 3 and exponents of 5, 7, and 12 that would make her answer correct.

Exercise 6

A number y satisfies $y^{24} - 256 = 0$. What equation does the number $x = y^4$ satisfy?

For any numbers x and y, and positive integer n,

$$(xy)^n = x^n y^n$$

because

$$\begin{aligned}(xy)^n &= \underbrace{(xy)\cdots(xy)}_{n\ times} \\ &= \underbrace{(x \cdot x \cdots x)}_{n\ times} \cdot \underbrace{(y \cdot y \cdots y)}_{n\ times} \\ &= x^n y^n\end{aligned}$$

Exercise 7

$(11 \times 4)^9 =$

Exercise 8

$(3^2 \times 7^4)^5 =$

Exercise 9

Let a, b, and c be numbers.

$(3^2 a^4)^5 =$

Exercise 10

Let x be a number.

$(5x)^7 =$

Exercise 11

Let x and y be numbers.

$(5xy^2)^7 =$

Exercise 12

Let a, b, and c be numbers.

$(a^2bc^3)^4 =$

Exercise 13

Let x and y be numbers, $y \neq 0$, and let n be a positive integer. How is $\left(\frac{x}{y}\right)^n$ related to x^n and y^n?

Problem Set

1. Show (prove) in detail why $(2 \cdot 3 \cdot 4)^4 = 2^4 3^4 4^4$.

2. Show (prove) in detail why $(xyz)^4 = x^4 y^4 z^4$ for any numbers x, y, z.

3. Show (prove) in detail why $(xyz)^n = x^n y^n z^n$ for any numbers x, y, z, and for any positive integer n.