Lesson 6: Finite and Infinite Decimals

Classwork

Exercises 1–5

1. Use long division to determine the decimal expansion of $\frac{54}{20}$.

2. Use long division to determine the decimal expansion of $\frac{7}{8}$.

3. Use long division to determine the decimal expansion of $\frac{8}{9}$.

4. Use long division to determine the decimal expansion of $\frac{22}{7}$.

5. What do you notice about the decimal expansions of Exercises 1 and 2 compared to the decimal expansions of Exercises 3 and 4?

Example 1

Consider the fraction $\frac{5}{8}$. Is it equal to a finite decimal? How do you know?

Example 2

Consider the fraction $\frac{17}{125}$. Is it equal to a finite or infinite decimal? How do you know?

Exercises 6–10

Show your steps, but use a calculator for the multiplications.

6. Convert the fraction $\frac{7}{8}$ to a decimal.

   a. Write the denominator as a product of 2's or 5's. Explain why this way of rewriting the denominator helps to find the decimal representation of $\frac{7}{8}$.

   b. Find the decimal representation of $\frac{7}{8}$. Explain why your answer is reasonable.

7. Convert the fraction $\frac{43}{64}$ to a decimal.

8. Convert the fraction $\frac{29}{125}$ to a decimal.

9. Convert the fraction $\frac{19}{34}$ to a decimal.

10. Identify the type of decimal expansion for each of the numbers in Exercises 6–9 as finite or infinite. Explain why their decimal expansion is such.

Example 3

Write $\frac{7}{80}$ as a decimal. Will it be finite or infinite? Explain.

Example 4

Write $\frac{3}{160}$ as a decimal. Will it be finite or infinite? Explain.

Exercises 11–13

Show your steps, but use a calculator for the multiplications.

11. Convert the fraction $\frac{37}{40}$ to a decimal.

    a. Write the denominator as a product of 2's and/or 5's. Explain why this way of rewriting the denominator helps to find the decimal representation of $\frac{37}{40}$.

    b. Find the decimal representation of $\frac{37}{40}$. Explain why your answer is reasonable.

12. Convert the fraction $\frac{3}{250}$ to a decimal.

13. Convert the fraction $\frac{7}{1{,}250}$ to a decimal.

Lesson Summary

Fractions with denominators that can be expressed as products of 2's and/or 5's have decimal expansions that are finite.

Example:

Does the fraction $\frac{1}{8}$ have a finite or infinite decimal expansion?

Since $8 = 2^3$, then the fraction has a finite decimal expansion. The decimal expansion is found by:

$$\frac{1}{8} = \frac{1}{2^3} = \frac{1 \times 5^3}{2^3 \times 5^3} = \frac{125}{10^3} = 0.125$$

When the denominator of a fraction cannot be expressed as a product of 2's and/or 5's then the decimal expansion of the number will be infinite.

When infinite decimals repeat, such as $0.8888888\ldots$ or $0.454545454545\ldots$, they are typically abbreviated using the notation $0.\overline{8}$ and $0.\overline{45}$, respectively. The notation indicates that the digit 8 repeats indefinitely and that the two-digit block 45 repeats indefinitely.

Problem Set

Convert each fraction to a finite decimal. If the fraction cannot be written as a finite decimal, then state how you know. Show your steps, but use a calculator for the multiplications.

1. $\frac{2}{32}$

2. $\frac{99}{125}$

   a. Write the denominator as a product of 2's and/or 5's. Explain why this way of rewriting the denominator helps to find the decimal representation of $\frac{99}{125}$.

   b. Find the decimal representation of $\frac{99}{125}$. Explain why your answer is reasonable.

3. $\frac{15}{128}$

4. $\frac{8}{15}$

5. $\frac{3}{28}$

6. $\frac{13}{400}$

7. $\frac{5}{64}$

8. $\frac{15}{35}$

9. $\frac{199}{250}$

10. $\frac{219}{625}$