Lesson 11: The Decimal Expansion of Some Irrational Numbers

Classwork

Opening Exercise

Place $\sqrt{28}$ on a number line. What decimal do you think $\sqrt{28}$ is equal to? Explain your reasoning.

Example 1

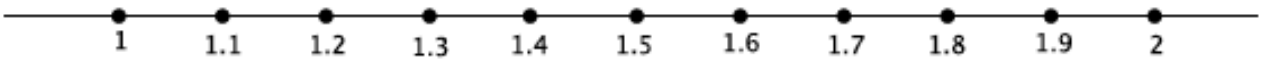
Recall the Basic Inequality:

Let c and d be two positive numbers, and let n be a fixed positive integer. Then $c < d$ if and only if $c^n < d^n$.

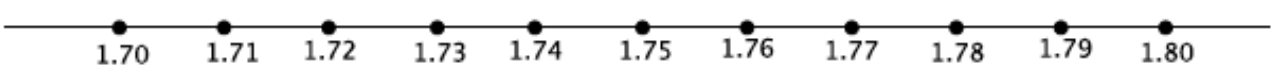
Write the decimal expansion of $\sqrt{3}$.

First approximation:

Second approximation:



Third approximation:

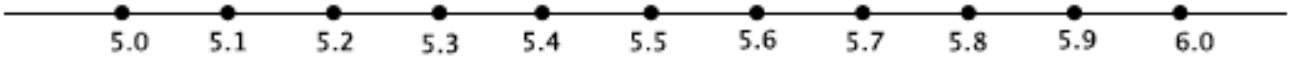


Example 2

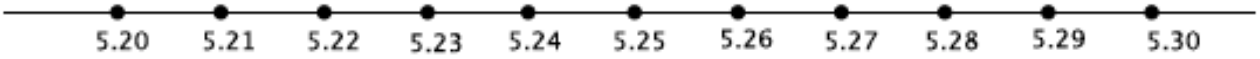
Write the decimal expansion of $\sqrt{28}$.

First approximation:

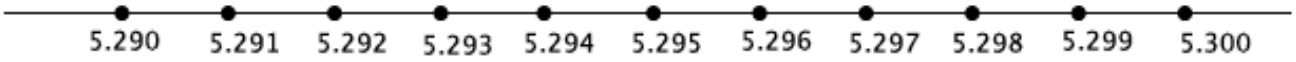
Second approximation:



Third approximation:



Fourth approximation:



Exercise 2

Between which interval of hundredths would $\sqrt{14}$ be located? Show your work.

Lesson Summary

To get the decimal expansion of a square root of a non-perfect square you must use the method of rational approximation. Rational approximation is a method that uses a sequence of rational numbers to get closer and closer to a given number to estimate the value of the number. The method requires that you investigate the size of the number by examining its value for increasingly smaller powers of 10 (i.e., tenths, hundredths, thousandths, and so on). Since $\sqrt{22}$ is not a perfect square, you would use rational approximation to determine its decimal expansion.

Example:

Begin by determining which two integers the number would lie.

$\sqrt{22}$ is between the integers 4 and 5 because $4^2 < \left(\sqrt{22}\right)^2 < 5^2$, which is equal to $16 < 22 < 25$.

Next, determine which interval of tenths the number belongs.

$\sqrt{22}$ is between 4.6 and 4.7 because $4.6^2 < \left(\sqrt{22}\right)^2 < 4.7^2$, which is equal to $21.16 < 22 < 22.09$.

Next, determine which interval of hundredths the number belongs.

$\sqrt{22}$ is between 4.69 and 4.70 because $4.69^2 < \left(\sqrt{22}\right)^2 < 4.70^2$, which is equal to $21.9961 < 22 < 22.09$.

A good estimate of the value of $\sqrt{22}$ is 4.69 because 22 is closer to 21.9961 than it is to 22.09.

Notice that with each step we are getting closer and closer to the actual value, 22. This process can continue using intervals of thousandths, ten-thousandths, and so on.

Any number that cannot be expressed as a rational number is called an *irrational number*. Irrational numbers are those numbers with decimal expansions that are infinite and do not have a repeating block of digits.

Problem Set

1. Use the method of rational approximation to determine the decimal expansion of $\sqrt{84}$. Determine which interval of hundredths it would lie in.
2. Get a 3 decimal digit approximation of the number $\sqrt{34}$.
3. Write the decimal expansion of $\sqrt{47}$ to at least 2 decimal digits.
4. Write the decimal expansion of $\sqrt{46}$ to at least 2 decimal digits.
5. Explain how to improve the accuracy of decimal expansion of an irrational number.
6. Is the number $\sqrt{125}$ rational or irrational? Explain.
7. Is the number $0.646464646\ldots$ rational or irrational? Explain.
8. Is the number $3.741657387\ldots$ rational or irrational? Explain.
9. Is the number $\sqrt{99}$ rational or irrational? Explain.
10. Challenge: Get a 2 decimal digit approximation of the number $\sqrt[3]{9}$.