

Lesson 10: Linear Models

In previous lessons, you used data that follow a linear trend either in the positive direction or the negative direction and informally fitted a line through the data. You determined the equation of an informal fitted line and used it to make predictions.

In this lesson, you will use a function to model a linear relationship between two numerical variables and interpret the slope and intercept of the linear model in the context of the data. Recall that a function is a rule that relates a dependent variable to an independent variable.

In statistics, a dependent variable is also called a response variable or a predicted variable. An independent variable is also called an explanatory variable or a predictor variable.

Classwork

Example 1

Predicting the value of a numerical dependent (response) variable based on the value of a given numerical independent variable has many applications in statistics. The first step in the process is to be able to identify the dependent variable (predicted variable) and the independent variable (predictor).

There may be several independent variables that might be used to predict a given dependent variable. For example, suppose you want to predict how well you are going to do on an upcoming statistics quiz. One of the possible independent variables is how much time you put into studying for the quiz. What are some other possible numerical independent variables that could relate to how well you are going to do on the quiz?

Exercises 1–2

1. For each of the following dependent (response) variables, identify two possible numerical independent (explanatory) variables that might be used to predict the value of the dependent variable.

Response variable	Possible explanatory variables
Height of a son	
Number of points scored in a game by a basketball player	
Number of hamburgers to make for a family picnic	
Time it takes a person to run a mile	
Amount of money won by a contestant on Jeopardy (television game show)	

Response variable	Possible explanatory variables
Fuel efficiency (in miles per gallon) for a car	
Number of honey bees in a beehive at a particular time	
Number of blooms on a dahlia plant	
Number of forest fires in a state during a particular year	

2. Now, reverse your thinking. For each of the following numerical independent variables, write a possible numerical dependent variable.

Dependent variable	Possible independent variable
	Age of a student
	Height of a golfer
	Amount of a pain-reliever taken
	Number of years of education
	Amount of fertilizer used on a garden
	Size of a diamond in a ring
	Total salary for all of a team's players

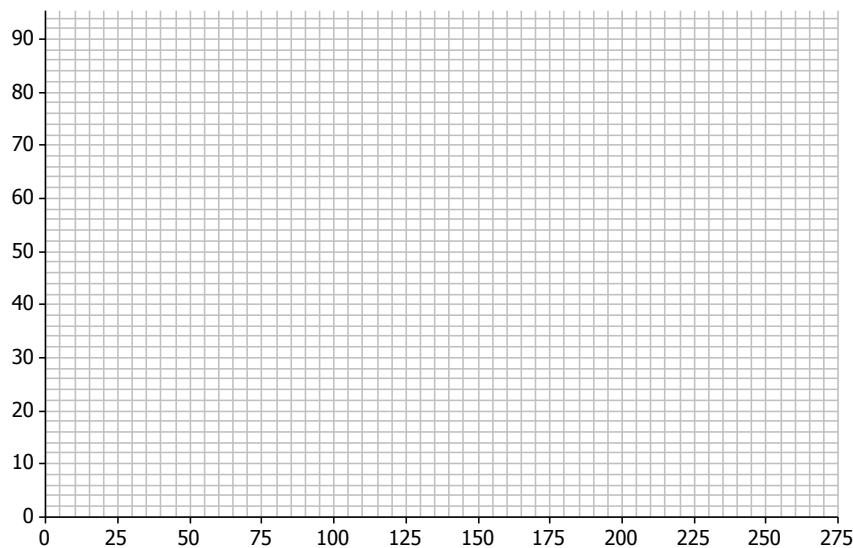
Example 2

A cell phone company is offering the following basic cell phone plan to its customers: A customer pays a monthly fee of \$40.00. In addition, the customer pays \$0.15 per text message sent from the cell phone. There is no limit to the number of text messages per month, and there is no charge for receiving text messages.

Exercises 3–9

3. Determine the following:
 - a. Justin never sends a text message. What would be his total monthly cost?
 - b. During a typical month, Abbey sends 25 text messages. What is her total cost for a typical month?
 - c. Robert sends at least 250 text messages a month. What would be an estimate of the least his total monthly cost is likely to be?
4. Write a linear model describing the relationship between the number of text messages sent and the total monthly cost using descriptive words.
5. Is the relationship between the number of text messages sent and the total monthly cost linear? Explain your answer.

6. Let x represent the independent variable and y represent the dependent variable. Write the function representing the relationship you indicated in Exercise 4 using the variables x and y .
7. Explain what \$ 0.15 represents in this relationship.
8. Explain what \$40.00 represents in this relationship.
9. Sketch a graph of this relationship on the following coordinate grid. Clearly label the axes and include units in the labels.



Exercise 10

10. LaMoynes needs four more pieces of lumber for his scout project. The pieces can be cut from one large piece of lumber according to the following pattern.



The lumberyard will make the cuts for LaMoynes at a fixed cost of \$2.25 plus an additional cost of 25 cents per cut. One cut is free.

- What is the functional relationship between the total cost of cutting a piece of lumber and the number of cuts required? What is the equation of this function? Be sure to define the variables in the context of this problem.
- Use the equation to determine LaMoynes's total cost for cutting.
- Interpret the slope of the equation in words in the context of this problem.
- Interpret the intercept of your equation in words in the context of this problem. Does interpreting the intercept make sense in this problem? Explain.

Exercise 11

11. Omar and Olivia were curious about the size of coins. They measured the diameter and circumference of several coins and found the following data.

US Coin	Diameter (mm)	Circumference (mm)
Penny	19.0	59.7
Nickel	21.2	66.6
Dime	17.9	56.2
Quarter	24.3	76.3
Half Dollar	30.6	96.1

- a. Wondering if there was any relationship between diameter and circumference, they thought about drawing a picture. Draw a scatter plot that displays circumference in terms of diameter.
- b. Do you think that circumference and diameter are related? Explain.
- c. Find the equation of the function relating circumference to the diameter of a coin.
- d. The value of the slope is approximately equal to the value of π . Explain why this makes sense.
- e. What is the value of the intercept? Explain why this makes sense.

Lesson Summary

- A linear functional relationship between a dependent and independent numerical variable has the form $y = mx + b$ or $y = a + bx$
- In statistics, a dependent variable is one that is predicted and an independent variable is the one that is used to make the prediction.
- The graph of a linear function describing the relationship between two variables is a line.

Problem Set

1. The Mathematics Club at your school is having a meeting. The advisor decides to bring bagels and his award-winning strawberry cream cheese. To determine his cost, from past experience he figures 1.5 bagels per student. A bagel costs 65 cents, and the special cream cheese costs \$3.85 and will be able to serve all of the anticipated students attending the meeting
 - a. Find an equation that relates his total cost to the number of students he thinks will attend the meeting.
 - b. Interpret the slope of the equation in words in the context of the problem.
 - c. Interpret the intercept of the equation in words in the context of the problem. Does interpreting the intercept make sense? Explain.
2. John, his daughter Dawn, and grandson Ron get together to exercise (walk/jog) for 45 minutes. John has arthritic knees but manages to walk $1\frac{1}{2}$ miles. Dawn walks $2\frac{1}{4}$ miles, while Ron manages to run 6 miles.
 - a. Draw an appropriate graph and connect the points to show that there is a linear relationship between the distance that each traveled based on how fast each traveled (speed). Note that the speed for a person who travels 3 miles in 45 minutes, or $\frac{3}{4}$ hours, is $3 \div \frac{3}{4} = 4$ miles per hour.
 - b. Find an equation that expresses distance in terms of speed (how fast one goes).
 - c. Interpret the slope of the equation in words in the context of the problem.
 - d. Interpret the intercept of the equation in words in the context of the problem. Does interpreting the intercept make sense? Explain.

3. Simple interest is money that is paid on a loan. Simple interest is calculated by taking the amount of the loan and multiplying it by the rate of interest per year and the number of years the loan is outstanding. For college, Jodie's older brother has taken out a student loan for \$4,500 at an interest rate of 5.6%, or 0.056. When he graduates in four years, he will have to pay back the loan amount plus interest for four years. Jodie is curious as to how much her brother will have to pay.
- Jodie claims that his brother will have to pay her a total of \$5,508. Do you agree? Explain. As an example, 8% simple interest on \$1,200 for one year is $(0.08)(1,200) = \$96$. The interest for two years would be $2 \times \$96$ or \$192.
 - Write an equation for the total cost to repay a loan of $\$P$ if the rate of interest for a year is r (expressed as a decimal) for a time span of t years.
 - If P and r are known, is the equation a linear equation?
 - Interpret the slope of the equation in words in the context of this problem.
 - Interpret the intercept of the equation in words in the context of this problem. Does interpreting the intercept make sense? Explain.