

Lesson 31: System of Equations Leading to Pythagorean Triples

Classwork

Exercises

1. Identify two Pythagorean triples using the known triple 3, 4, 5 (other than 6, 8, 10).
2. Identify two Pythagorean triples using the known triple 5, 12, 13.
3. Identify two triples using either 3, 4, 5 or 5, 12, 13.

Use the system $\begin{cases} x + y = \frac{t}{s} \\ x - y = \frac{s}{t} \end{cases}$ to find Pythagorean triples for the given values of s and t . Recall that the solution, in the form of $(\frac{c}{b}, \frac{a}{b})$, is the triple, a, b, c .

4. $s = 4, t = 5$

5. $s = 7, t = 10$

6. $s = 1, t = 4$

7. Use a calculator to verify that you found a Pythagorean triple in each of the Exercises 4–6. Show your work below.

Lesson Summary

A Pythagorean triple is a set of three positive integers that satisfies the equation $a^2 + b^2 = c^2$.

An infinite number of Pythagorean triples can be found by multiplying the numbers of a known triple by a whole number. For example, 3, 4, 5 is a Pythagorean triple. Multiply each number by 7, then you have 21, 28, 35 which is also a Pythagorean triple.

The system of linear equations, $\begin{cases} x + y = \frac{t}{s} \\ x - y = \frac{s}{t} \end{cases}$ can be used to find Pythagorean triples, just like the Babylonians did 4,000 years ago.

Problem Set

1. Explain in terms of similar triangles why it is that when you multiply the known Pythagorean triple 3, 4, 5 by 12, it generates a Pythagorean triple.
2. Identify three Pythagorean triples using the known triple 8, 15, 17.
3. Identify three triples (numbers that satisfy $a^2 + b^2 = c^2$, but a, b, c are not whole numbers) using the triple 8, 15, 17.

Use the system $\begin{cases} x + y = \frac{t}{s} \\ x - y = \frac{s}{t} \end{cases}$ to find Pythagorean triples for the given values of s and t . Recall that the solution, in the form of $(\frac{c}{b}, \frac{a}{b})$, is the triple, a, b, c .

4. $s = 2, t = 9$
5. $s = 6, t = 7$
6. $s = 3, t = 4$
7. Use a calculator to verify that you found a Pythagorean triple in each of the problems 4–6. Show your work below.