## Lesson 19: The Graph of a Linear Equation in Two Variables is a

## Line

## Classwork

## Exercises

Theorem: The graph of a linear equation $y=m x+b$ is a non-vertical line with slope $m$ and passing through $(0, b)$, where $b$ is a constant.

1. Prove the theorem by completing parts (a)-(c). Given two distinct points, $P$ and $Q$, on the graph of $y=m x+b$ and let $l$ be the line passing through $P$ and $Q$. You must show:
(1) any point on the graph of $y=m x+b$ is on line $l$, and
(2) any point on the line $l$ is on the graph of $y=m x+b$.
a. Proof of (1): Let $R$ be any point on the graph of $y=m x+k$. Show that $R$ is on $l$. Begin by assuming it is not. Assume the graph looks like the diagram below where $R$ is on $l^{\prime}$.


What is the slope of line $l ?$

What is the slope of line $l^{\prime}$ ?

What can you conclude about lines $l$ and $l^{\prime}$ ? Explain.
b. Proof of (2): Let $S$ be any point on line $l$, as shown.


Show that $S$ is a solution to $y=m x+b$. Hint: Use the point $(0, b)$.
c. Now that you have shown that any point on the graph of $y=m x+b$ is on line $l$ (part (a)), and any point on line $l$ is on the graph of $y=m x+b$ (part (b)), what can you conclude about the graphs of linear equations?
2. Use $x=4$ and $x=-4$ to find two solutions to the equation $x+2 y=6$. Plot the solutions as points on the coordinate plane and connect the points to make a line.
a. Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation $x+2 y=6$.
b. When $x=1$, what is the value of $y$ ? Does this solution appear to be a point on the line?
c. When $x=-3$, what is the value of $y$ ? Does this solution appear to be a point on the line?
d. Is the point $(3,2)$ on the line?
e. Is the point $(3,2)$ a solution to the linear equation $x+2 y=6$ ?
3. Use $x=4$ and $x=1$ to find two solutions to the equation $3 x-y=9$. Plot the solutions as points on the coordinate plane and connect the points to make a line.
a. Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation $3 x-y=9$.
b. When $x=4.5$, what is the value of $y$ ? Does this solution appear to be a point on the line?
c. When $x=\frac{1}{2}$, what is the value of $y$ ? Does this solution appear to be a point on the line?
d. Is the point $(2,4)$ on the line?
e. Is the point $(2,4)$ a solution to the linear equation $3 x-y=9$ ?
4. Use $x=3$ and $x=-3$ to find two solutions to the equation $2 x+3 y=12$. Plot the solutions as points on the coordinate plane and connect the points to make a line.
a. Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation $2 x+3 y=12$.
b. When $x=2$, what is the value of $y$ ? Does this solution appear to be a point on the line?
c. When $x=-2$, what is the value of $y$ ? Does this solution appear to be a point on the line?
d. Is the point $(8,-3)$ on the line?
e. Is the point $(8,-3)$ a solution to the linear equation $2 x+3 y=12$ ?
5. Use $x=4$ and $x=-4$ to find two solutions to the equation $x-2 y=8$. Plot the solutions as points on the coordinate plane and connect the points to make a line.
a. Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation $x-2 y=8$.
b. When $x=7$, what is the value of $y$ ? Does this solution appear to be a point on the line?
c. When $x=-3$, what is the value of $y$ ? Does this solution appear to be a point on the line?
d. Is the point $(-2,-3)$ on the line?
e. Is the point $(-2,-3)$ a solution to the linear equation $x-2 y=8$ ?
6. Based on your work in Exercises 2-5, what conclusions can you draw about the points on a line and solutions to a linear equation?
7. Based on your work in Exercises 2-5, will a point that is not a solution to a linear equation be a point on the graph of a linear equation? Explain.
8. Based on your work in Exercises 2-5, what conclusions can you draw about the graph of a linear equation?
9. Graph the equation $-3 x+8 y=24$ using intercepts.
10. Graph the equation $x-6 y=15$ using intercepts.
11. Graph the equation $4 x+3 y=21$ using intercepts.

## Lesson Summary

The graph of a linear equation is a line. A linear equation can be graphed using two-points: the $x$-intercept and the $y$-intercept.

Example:
Graph the equation: $2 x+3 y=9$.
Replace $x$ with zero and solve for $y$ to determine the $y$-intercept:

$$
\begin{array}{r}
2(0)+3 y=9 \\
3 y=9 \\
y=3
\end{array}
$$

The $y$-intercept is at $(0,3)$.
Replace $y$ with zero and solve for $x$ to determine the $x$-intercept:

$$
\begin{aligned}
2 x+3(0) & =9 \\
2 x & =9 \\
x & =\frac{9}{2}
\end{aligned}
$$

The $x$-intercept is at $\left(\frac{9}{2}, 0\right)$.


## Problem Set

Graph each of the equations in the Problem Set on a different pair of $x$ and $y$ axes.

1. Graph the equation: $y=-6 x+12$.
2. Graph the equation: $9 x+3 y=18$.
3. Graph the equation: $y=4 x+2$.
4. Graph the equation: $y=-\frac{5}{7} x+4$.
5. Graph the equation: $\frac{3}{4} x+y=8$.
6. Graph the equation: $2 x-4 y=12$.
7. Graph the equation: $y=3$. What is the slope of the graph of this line?
8. Graph the equation: $x=-4$. What is the slope of the graph of this line?
9. Is the graph of $4 x+5 y=\frac{3}{7}$ a line? Explain.
10. Is the graph of $6 x^{2}-2 y=7$ a line? Explain.
