Lesson 19: The Graph of a Linear Equation in Two Variables is a

Line

Classwork

Exercises

Theorem: The graph of a linear equation y = mx + b is a non-vertical line with slope m and passing through (0, b), where b is a constant.

- 1. Prove the theorem by completing parts (a)–(c). Given two distinct points, P and Q, on the graph of y = mx + b and let l be the line passing through P and Q. You must show:
 - (1) any point on the graph of y = mx + b is on line *l*, and
 - (2) any point on the line *l* is on the graph of y = mx + b.
 - a. Proof of (1): Let R be any point on the graph of y = mx + k. Show that R is on l. Begin by assuming it is not. Assume the graph looks like the diagram below where R is on l'.



What is the slope of line *l*?

What is the slope of line l'?

What can you conclude about lines l and l'? Explain.

b. Proof of (2): Let *S* be any point on line *l*, as shown.



Show that *S* is a solution to y = mx + b. Hint: Use the point (0, b).

c. Now that you have shown that any point on the graph of y = mx + b is on line l (part (a)), and any point on line l is on the graph of y = mx + b (part (b)), what can you conclude about the graphs of linear equations?

- 2. Use x = 4 and x = -4 to find two solutions to the equation x + 2y = 6. Plot the solutions as points on the coordinate plane and connect the points to make a line.
 - a. Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation x + 2y = 6.

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- b. When x = 1, what is the value of y? Does this solution appear to be a point on the line?
- c. When x = -3, what is the value of y? Does this solution appear to be a point on the line?
- d. Is the point (3, 2) on the line?
- e. Is the point (3, 2) a solution to the linear equation x + 2y = 6?
- 3. Use x = 4 and x = 1 to find two solutions to the equation 3x y = 9. Plot the solutions as points on the coordinate plane and connect the points to make a line.
 - a. Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation 3x y = 9.

b. When x = 4.5, what is the value of y? Does this solution appear to be a point on the line?

- c. When $x = \frac{1}{2}$, what is the value of y? Does this solution appear to be a point on the line?
- d. Is the point (2, 4) on the line?
- e. Is the point (2, 4) a solution to the linear equation 3x y = 9?
- 4. Use x = 3 and x = -3 to find two solutions to the equation 2x + 3y = 12. Plot the solutions as points on the coordinate plane and connect the points to make a line.
 - a. Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation 2x + 3y = 12.

- b. When x = 2, what is the value of y? Does this solution appear to be a point on the line?
- c. When x = -2, what is the value of y? Does this solution appear to be a point on the line?
- d. Is the point (8, -3) on the line?
- e. Is the point (8, -3) a solution to the linear equation 2x + 3y = 12?

- 5. Use x = 4 and x = -4 to find two solutions to the equation x 2y = 8. Plot the solutions as points on the coordinate plane and connect the points to make a line.
 - a. Identify two other points on the line with integer coordinates. Verify that they are solutions to the equation x 2y = 8.

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- b. When x = 7, what is the value of y? Does this solution appear to be a point on the line?
- c. When x = -3, what is the value of y? Does this solution appear to be a point on the line?
- d. Is the point (-2, -3) on the line?
- e. Is the point (-2, -3) a solution to the linear equation x 2y = 8?
- 6. Based on your work in Exercises 2–5, what conclusions can you draw about the points on a line and solutions to a linear equation?
- 7. Based on your work in Exercises 2–5, will a point that is not a solution to a linear equation be a point on the graph of a linear equation? Explain.
- 8. Based on your work in Exercises 2–5, what conclusions can you draw about the graph of a linear equation?

- 9. Graph the equation -3x + 8y = 24 using intercepts.
- 10. Graph the equation x 6y = 15 using intercepts.
- 11. Graph the equation 4x + 3y = 21 using intercepts.

Lesson Summary

The graph of a linear equation is a line. A linear equation can be graphed using two-points: the *x*-intercept and the *y*-intercept.

Example:

Graph the equation: 2x + 3y = 9.

Replace *x* with zero and solve for *y* to determine the *y*-intercept:

2(0) + 3y = 93y = 9y = 3

The y-intercept is at (0, 3).

Replace y with zero and solve for x to determine the x-intercept:

$$2x + 3(0) = 9$$
$$2x = 9$$
$$x = \frac{9}{2}$$

The *x*-intercept is at $\left(\frac{9}{2}, 0\right)$.



Problem Set

Graph each of the equations in the Problem Set on a different pair of x and y axes.

- 1. Graph the equation: y = -6x + 12.
- 2. Graph the equation: 9x + 3y = 18.
- 3. Graph the equation: y = 4x + 2.
- 4. Graph the equation: $y = -\frac{5}{7}x + 4$.

- 5. Graph the equation: $\frac{3}{4}x + y = 8$.
- 6. Graph the equation: 2x 4y = 12.
- 7. Graph the equation: y = 3. What is the slope of the graph of this line?
- 8. Graph the equation: x = -4. What is the slope of the graph of this line?
- 9. Is the graph of $4x + 5y = \frac{3}{7}$ a line? Explain.
- 10. Is the graph of $6x^2 2y = 7$ a line? Explain.