Lesson 5: Identical Triangles

Classwork

Opening

When studying triangles, it is essential to be able to communicate about the parts of a triangle without any confusion. The following terms are used to identify particular angles or sides:

- between
- adjacent to
- opposite to
- the included [side/angle]

Opening Exercises 1–7

Use the figure $\triangle ABC$ to fill in the following blanks.

1. $\angle A$ is ____________________ sides AB and AC.
2. $\angle B$ is ____________________ to side AB and to side BC.
3. Side AB is ____________________ to $\angle C$.
4. Side ______ is the included side of $\angle B$ and $\angle C$.
5. $\angle$______ is opposite to Side AC.
6. Side AB is between angles $\angle$______ and $\angle$______.
7. What is the included angle of AB and BC?

Now that we know what to call the parts within a triangle, we consider how to discuss two triangles. We need to compare the parts of the triangles in a way that is easy to understand. To establish some alignment between the triangles, the vertices of the two triangles are paired up. This is called a correspondence. Specifically, a correspondence between two triangles is a pairing of each vertex of one triangle with one (and only one) vertex of the other triangle. A correspondence provides a systematic way to compare parts of two triangles.

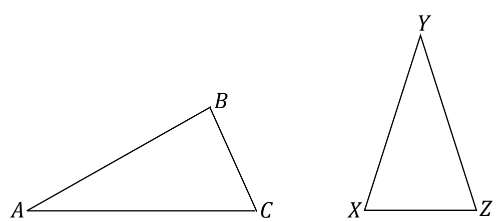
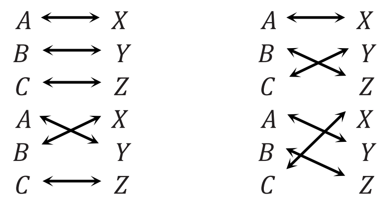


Figure 1

In Figure 1, we can choose to assign a correspondence so that A matches to X, B matches to Y, and C matches to Z. We notate this correspondence with double-arrows: $A \leftrightarrow X$, $B \leftrightarrow Y$, and $C \leftrightarrow Z$. This is just one of six possible correspondences between the two triangles. Four of the six correspondences are listed below; find the remaining two correspondences.



A simpler way to indicate the triangle correspondences is to let the order of the vertices define the correspondence; i.e., the first corresponds to the first, the second to the second, and the third to the third. The correspondences above can be written in this manner. Write the remaining two correspondences in this way.

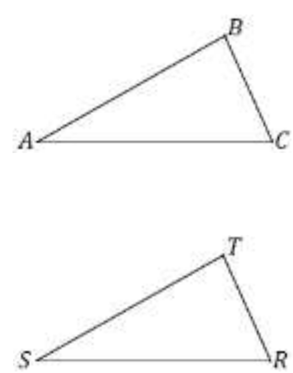
$\triangle ABC \leftrightarrow \triangle XYZ$	$\triangle ABC \leftrightarrow \triangle XZY$
$\triangle ABC \leftrightarrow \triangle YXZ$	$\triangle ABC \leftrightarrow \triangle YZX$

With a correspondence in place, comparisons can be made about corresponding sides and corresponding angles. The following are corresponding vertices, angles, and sides for the triangle correspondence $\triangle ABC \leftrightarrow \triangle YXZ$. Complete the missing correspondences:

Vertices:	$A \leftrightarrow Y$	$B \leftrightarrow$	$C \leftrightarrow$
Angles:	$\angle A \leftrightarrow \angle Y$	$\angle B \leftrightarrow$	$\angle C \leftrightarrow$
Sides:	$AB \leftrightarrow YX$	$BC \leftrightarrow$	$CA \leftrightarrow$

Example 1

Triangle Correspondence	$\triangle ABC \leftrightarrow \triangle STR$
Correspondence of Vertices	
Correspondence of Angles	
Correspondence of Sides	



Examine Figure 2. By simply looking, it is impossible to tell the two triangles apart unless they are labeled. They look exactly the same (just as identical twins look the same). One triangle could be picked up and placed on top of the other.

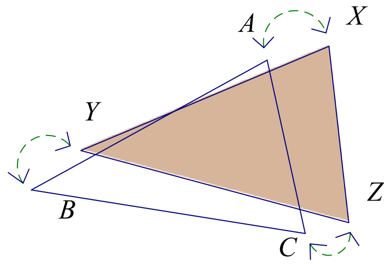
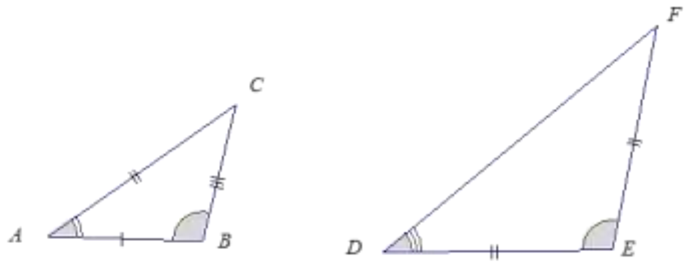


Figure 2

Two triangles are identical if there is a triangle correspondence so that corresponding sides and angles of each triangle is equal in measurement. In Figure 2, there is a correspondence that will match up equal sides and equal angles, $\triangle ABC \leftrightarrow \triangle XYZ$; we can conclude that $\triangle ABC$ is identical to $\triangle XYZ$. This is not to say that we cannot find a correspondence in Figure 2 so that unequal sides and unequal angles are matched up, but there certainly is one correspondence that will match up angles with equal measurements and sides of equal lengths, making the triangles identical.

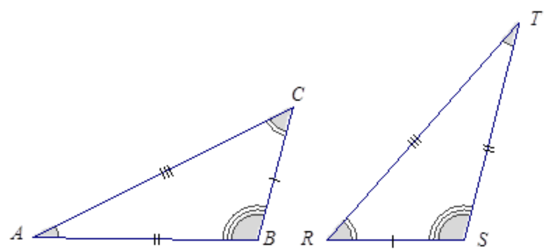
In discussing identical triangles, it is useful to have a way to indicate those sides and angles that are equal. We mark sides with tick marks and angles with arcs if we want to draw attention to them. If two angles or two sides have the same number of marks, it means they are equal.



In this figure, $AC = DE = EF$, and $\angle B = \angle E$.

Example 2

Two identical triangles are shown below. Give a triangle correspondence that matches equal sides and equal angles.



Exercise 8

Sketch two triangles that have a correspondence. Describe the correspondence in symbols or words. Have a partner check your work.

Problem Set

Given the following triangle correspondences, use double arrows to show the correspondence between vertices, angles, and sides.

1.

Triangle Correspondence	$\triangle ABC \leftrightarrow \triangle RTS$
Correspondence of Vertices	
Correspondence of Angles	
Correspondence of Sides	

2.

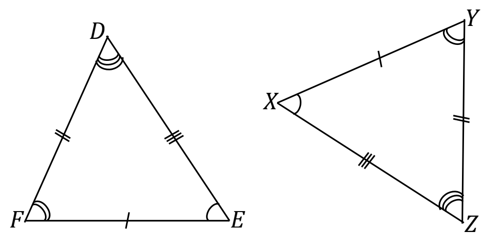
Triangle Correspondence	$\triangle ABC \leftrightarrow \triangle FGE$
Correspondence of Vertices	
Correspondence of Angles	
Correspondence of Sides	

3.

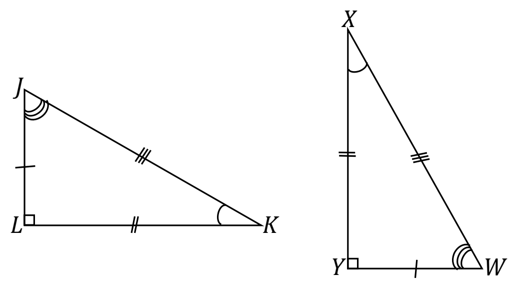
Triangle Correspondence	$\triangle QRP \leftrightarrow \triangle WYX$
Correspondence of Vertices	
Correspondence of Angles	
Correspondence of Sides	

Name the angle pairs and side pairs to find a triangle correspondence that matches sides of equal length and angles of equal angles measurements.

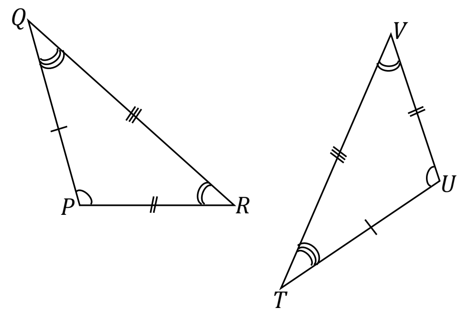
4.



5.

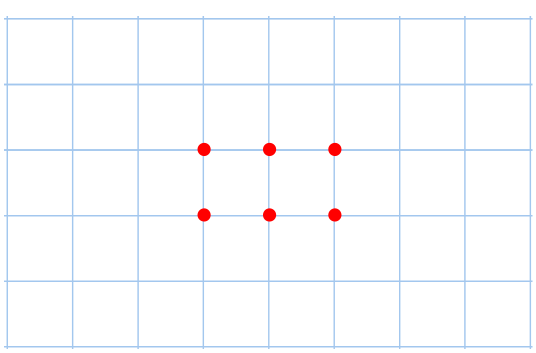


6.



7. Consider the following points in the coordinate plane.

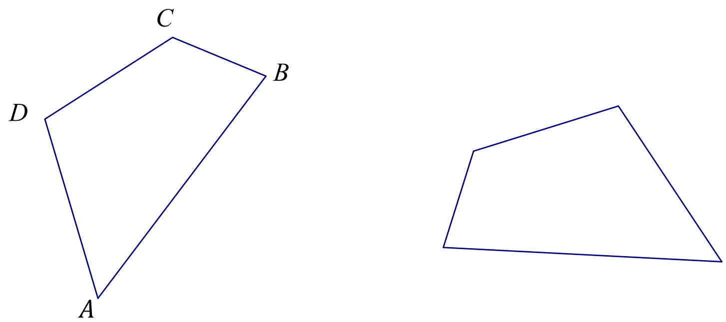
   a. How many different (non-identical) triangles can be drawn using any three of these six points as vertices?



   b. How can we be sure that there are no more possible triangles?

8. Quadrilateral $ABCD$ is identical with Quadrilateral $WXYZ$ with a correspondence $A \leftrightarrow W$, $B \leftrightarrow X$, $C \leftrightarrow Y$, and $D \leftrightarrow Z$.

   a. In the figure below, label points W, X, Y, and Z on the second quadrilateral.



   b. Set up a correspondence between the side lengths of the two quadrilaterals that matches sides of equal length.

   c. Set up a correspondence between the angles of the two quadrilaterals that matches angles of equal measure.