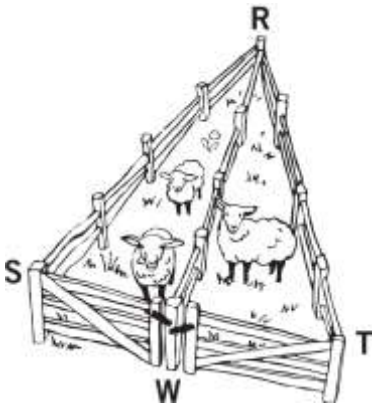
Lesson 15: Using Unique Triangles to Solve Real-World and Mathematical Problems

Classwork

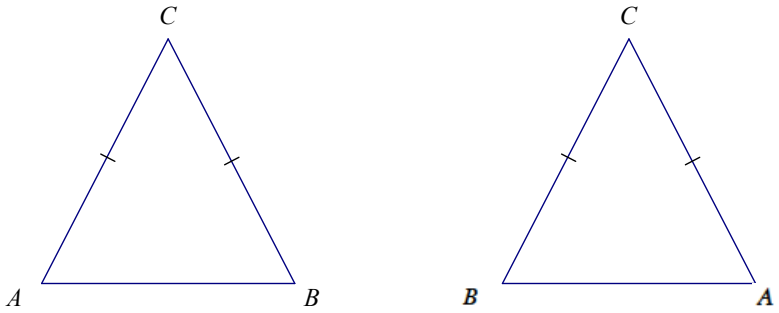
Example 1

A triangular fence with two equal angles, $\angle S = \angle T$, is used to enclose some sheep. A fence is constructed inside the triangle that exactly cuts the other angle into two equal angles: $\angle SRW = \angle TRW$. Show that the gates, represented by SW and WT, are the same width.



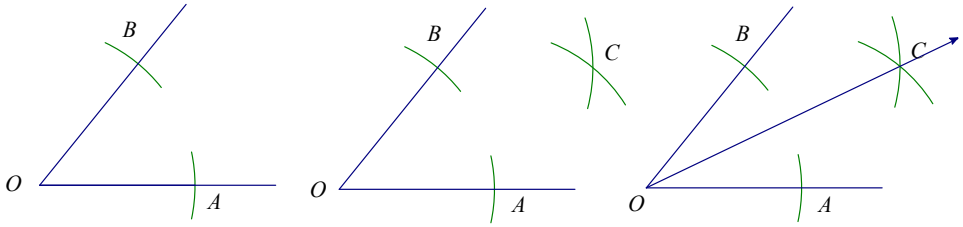
Example 2

In $\triangle ABC$, $AC = BC$. John says that the triangle correspondence $\triangle ABC \leftrightarrow \triangle BAC$ matches two sides and the included angle and shows that $\angle A = \angle B$. Is John correct?

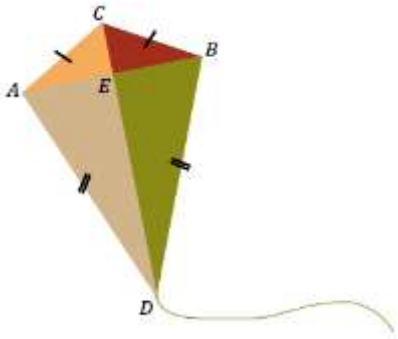


Exercises 1–4

1. Mary puts the center of her compass at the vertex O of the angle and locates points A and B on the sides of the angle. Next, she centers her compass at each of A and B to locate point C. Finally, she constructs the ray $\overrightarrow{OC}$. Explain why $\angle BOC = \angle AOC$.



2. Quadrilateral $ACBD$ is a model of a kite. The diagonals AB and CD represent the sticks that help keep the kite rigid.

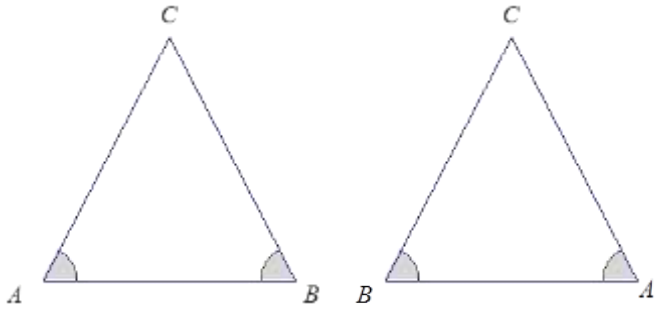


   a. John says that $\angle ACD = \angle BCD$. Can you use identical triangles to show that John is correct?

   b. Jill says that the two sticks are perpendicular to each other. Use the fact that $\angle ACD = \angle BCD$ and what you know about identical triangles to show $\angle AEC = 90°$.

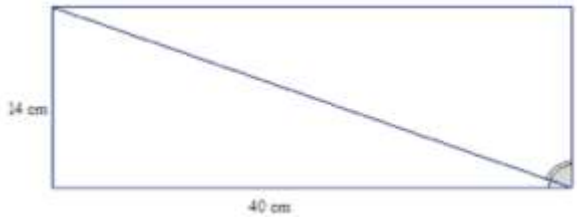
c. John says that Jill's triangle correspondence that shows the sticks are perpendicular to each other also shows that the sticks cross at the midpoint of the horizontal stick. Is John correct? Explain.

3. In $\triangle ABC$, $\angle A = \angle B$. Jill says that the triangle correspondence $\triangle ABC \leftrightarrow \triangle BAC$ matches two sides and the included angle and shows that $AC = BC$. Is Jill correct?



4. Right triangular corner flags are used to mark a soccer field. The vinyl flags have a base of 40 cm and a height of 14 cm.

a. Mary says that the two flags can be obtained by cutting a rectangle that is 40 cm × 14 cm on the diagonal. Will that create two identical flags? Explain.

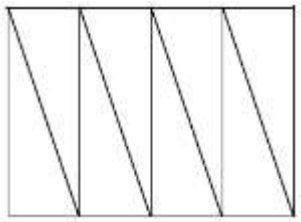


b. Will measures the two non-right angles on a flag and adds the measurements together. Can you explain, without measuring the angles, why his answer is 90°?

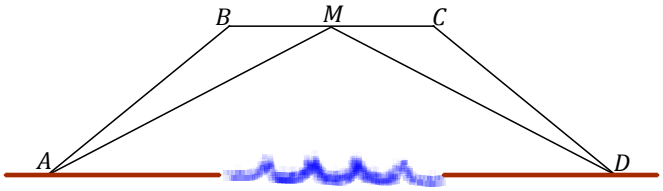
Problem Set

1. Jack is asked to cut a cake into 8 equal pieces. He first cuts it into equal fourths in the shape of rectangles, and then he cuts each rectangle along a diagonal.

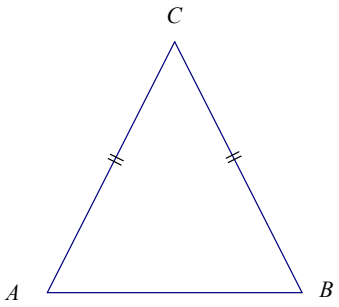
   Did he cut the cake into 8 equal pieces? Explain.



2. The bridge below, which crosses a river, is built out of two triangular supports. The point M lies on segment BC. The beams represented by AM and DM are equal in length, and the beams represented by AB and DC are equal in length. If the supports were constructed so that $\angle A$ and $\angle D$ are equal in measurement, is point M the midpoint of BC? Explain.



3. In $\triangle ABC$, $AC = BC$. Bill says that triangle correspondence $\triangle ABC \leftrightarrow \triangle BAC$ matches three equal sides and shows that $\angle A = \angle B$. Is Bill correct?



4. In the previous problem, Jill says that triangle correspondence $\triangle ABC \leftrightarrow \triangle BAC$ matches two equal sides and the included angle. This also shows that $\angle A = \angle B$. Is Jill correct?