Lesson 11: Conditions on Measurements that Determine a Triangle

Classwork

Exploratory Challenge 1

a. Can any three side lengths form a triangle? Why or why not?

b. Draw a triangle according to these instructions:
   - ✓ Draw segment AB of length 10 cm in your notebook.
   - ✓ Draw segment BC of length 3 cm on one piece of patty paper.
   - ✓ Draw segment AC of length 5 cm on the other piece of patty paper.
   - ✓ Line up the appropriate endpoint on each piece of patty paper with the matching endpoint on AB.
   - ✓ Use your pencil point to hold each patty paper in place, and adjust the paper to form $\triangle ABC$.

c. What do you notice?

d. What must be true about the sum of the lengths of AC and BC if the two segments were to just meet? Use your patty paper to verify your answer.

e. Based on your conclusion for part (c), what if $BC = 3$ cm as you originally had, but $AC = 10$ cm in length. Could you form $\triangle ABC$?

f. What must be true about the sum of the lengths of AC and BC if the two segments were to meet and form a triangle?

Exercise 1

Two sides of $\triangle DEF$ have lengths of 5 cm and 8 cm. What are all the possible whole-number lengths for the remaining side?

Exploratory Challenge 2

a. Which of the following conditions determine a triangle? Follow the instructions to try and draw $\triangle ABC$. Segment AB has been drawn for you as a starting point in each case.

i. Choose measurements of $\angle A$ and $\angle B$ for $\triangle ABC$ so that the sum of measurements is greater than $180°$. Label your diagram.

Your chosen angle measurements: $\angle A =$ $\angle B =$

Were you able to form a triangle? Why or why not?

A B

ii. Choose measurements of $\angle A$ and $\angle B$ for $\triangle ABC$ so that the measurement of $\angle A$ is supplementary to the measurement of $\angle B$. Label your diagram.

Your chosen angle measurements: $\angle A =$ $\angle B =$

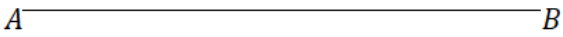
Were you able to form a triangle? Why or why not?

A B

iii. Choose measurements of $\angle A$ and $\angle B$ for $\triangle ABC$ so that the sum of measurements is less than $180°$. Label your diagram.

Your chosen angle measurements: $\angle A =$ $\angle B =$

Were you able to form a triangle? Why or why not?



b. Which condition must be true regarding angle measurements in order to determine a triangle?

c. Measure and label the formed triangle in part (b) with all three side lengths and the angle measurement for $\angle C$. Now, use a protractor, ruler, and compass to draw $\triangle A'B'C'$ with the same angle measurements, but side lengths that are half as long.

d. Do the three angle measurements of a triangle determine a unique triangle? Why or why not?

Exercise 2

Which of the following sets of angle measurements determines a triangle?

a. $30°, 120°$

b. $125°, 55°$

c. $105°, 80°$

d. $90°, 89°$

e. $91°, 89°$

Choose one example from above that does determine a triangle and one that does not. For each, explain why it does or does not determine a triangle using words and a diagram.

Problem Set

1. Decide whether each set of three given lengths determines a triangle. For any set of lengths that does determine a triangle, use a ruler and compass to draw the triangle. Label all side lengths. For sets of lengths that do not determine a triangle, write "Does not determine a triangle," and justify your response.
   a. 3 cm, 4 cm, 5 cm
   b. 1 cm, 4 cm, 5 cm
   c. 1 cm, 5 cm, 5 cm
   d. 8 cm, 3 cm, 4 cm
   e. 8 cm, 8 cm, 4 cm
   f. 4 cm, 4 cm, 4 cm

2. For each angle measurement below, provide one angle measurement that will determine a triangle and one that will not determine a triangle. Provide a brief justification for the angle measurements that will not form a triangle. Assume that the angles are being drawn to a horizontal segment AB; describe the position of the non-horizontal rays of angles $\angle A$ and $\angle B$.

$\angle A$	$\angle B$: A Measurement that Determines a Triangle	$\angle B$: A Measurement that Doesn't Determine a Triangle	Justification for No Triangle
40°			
100°			
90°			
135°			

3. For the given side lengths, provide the minimum and maximum whole-number side lengths that determine a triangle.

Given Side Lengths	Minimum Whole Number Third Side Length	Maximum Whole Number Third Side Length
5 cm, 6 cm		
3 cm, 7 cm		
4 cm, 10 cm		
1 cm, 12 cm		