

## Lesson 9: Comparing Estimated Probabilities to Probabilities

### Predicted by a Model

#### Game Show—Picking Blue!

Imagine, for a moment, the following situation: You and your classmates are contestants on a quiz show called *Picking Blue!* There are two bags in front of you, Bag A and Bag B. Each bag contains red and blue chips. You are told that one of the bags has exactly the same number of blue chips as red chips. But you are told nothing about the number of blue and red chips in the second bag.

Each student in your class will be asked to select either Bag A or Bag B. Starting with Bag A, a chip is randomly selected from the bag. If a blue chip is drawn, all of the students in your class who selected Bag A win a Blue Token. The chip is put back in the bag. After mixing up the chips in the bag, another chip is randomly selected from the bag. If the chip is blue, the students who picked Bag A win another Blue Token. After the chip is placed back into the bag, the process continues until a red chip is picked. When a red chip is picked, the game moves to Bag B. A chip from the Bag B is then randomly selected. If it is blue, all of the students who selected Bag B win a Blue Token. But if the chip is red, the game is over. Just like for Bag A, if the chip is blue, the process repeats until a red chip is picked from the bag. When the game is over, the students with the greatest number of Blue Tokens are considered the winning team.

Without any information about the bags, you would probably select a bag simply by guessing. But surprisingly, the show's producers are going to allow you to do some research before you select a bag. For the next 20 minutes, you can pull a chip from either one of the two bags, look at the chip, and then put the chip back in the bag. You can repeat this process as many times as you want within the 20 minutes. At the end of 20 minutes, you must make your final decision and select which of the bags you want to use in the game.

#### Getting Started

Assume that the producers of the show do not want give away a lot of their Blue Tokens. As a result, if one bag has the same number of red and blue chips, do you think the other bag would have more, or fewer, blue chips than red chips? Explain your answer.

### Planning the Research

Your teacher will provide you with two bags labeled A and B. You have 20 minutes to experiment with pulling chips one at a time from the bags. After you examine a chip, you must put it back in the bag. Remember, no fair peeking in the bags as that will disqualify you from the game. You can pick chips from just one bag, or you can pick chips from one bag and then the other bag.

Use the results from 20 minutes of research to determine which bag you will choose for the game.

Provide a description outlining how you will carry out your research:

### Carrying Out the Research

Share your plan with your teacher. Your teacher will verify whether your plan is within the rules of the quiz show. Approving your plan does not mean, however, that your teacher is indicating that your research method offers the most accurate way to determine which bag to select. If your teacher approves your research, carry out your plan as outlined. Record the results from your research, as directed by your teacher.

### Playing the Game

After the research has been conducted, the competition begins. First, your teacher will shake up Bag A. A chip is selected. If the chip is blue, all students who selected Bag A win an imaginary Blue Token. The chip is put back in the bag, and the process continues. When a red chip is picked from Bag A, students selecting Bag A have completed the competition. Your teacher will now shake up Bag B. A chip is selected. If it is blue, all students who selected Bag B win an imaginary Blue Token. The process continues until a red chip is picked. At that point, the game is over.

How many Blue Tokens did you win?

**Examining Your Results**

At the end of the game, your teacher will open the bags and reveal how many blue and red chips were in each bag. Answer the following questions. (See: *Closing Questions*.) After you have answered these questions, discuss them with your class.

1. Before you played the game, what were you trying to learn about the bags from your research?
2. What did you expect to happen when you pulled chips from the bag with the same number of blue and red chips? Did the bag that you thought had the same number of blue and red chips yield the results you expected?
3. How confident were you in predicting which bag had the same number of blue and red chips? Explain.
4. What bag did you select to use in the competition and why?
5. If you were the show's producers, how would you make up the second bag? (Remember, one bag has the same number of red and blue chips.)
6. If you picked a chip from Bag B 100 times and found that you picked each color exactly 50 times, would you know for sure that that bag was the one with equal numbers of each color?

**Lesson Summary:**

- The long-run relative frequencies can be used as estimated probabilities of events.
- Collecting data on a game or chance experiment is one way to estimate the probability of an outcome.
- The more data collected on the outcomes from a game or chance experiment, the closer the estimates of the probabilities are likely to be the actual probabilities.

**Problem Set**

Jerry and Michael played a game similar to *Picking Blue*. The following results are from their research using the same two bags:

Jerry’s research:

	Number of Red chips picked	Number of Blue chips picked
Bag A	2	8
Bag B	3	7

Michael’s research:

	Number of Red chips picked	Number of Blue chips picked
Bag A	28	12
Bag B	22	18

1. If all you knew about the bags were the results of Jerry’s research, which bag would you select for the game? Explain your answer.
2. If all you knew about the bags were the results of Michael’s research, which bag would you select for the game? Explain your answer.
3. Does Jerry’s research or Michael’s research give you a better indication of the make-up of the blue and red chips in each bag? Explain why you selected this research.
4. Assume there are 12 chips in each bag. Use either Jerry’s or Michael’s research to estimate the number of red and blue chips in each bag. Then explain how you made your estimates.

**Bag A**

Number of red chips:

Number of blue chips:

**Bag B**

Number of red chips:

Number of blue chips:

5. In a different game of *Picking Blue!*, two bags each contain red, blue, green, and yellow chips. One bag contains the same number of red, blue, green, and yellow chips. In the second bag, half the chips are blue. Describe a plan for determining which bag has more blue chips than any of the other colors.