## Lesson 26: Volume and Surface Area

## Classwork

## Opening Exercise

Explain to your partner how you would calculate the area of the shaded region. Then, calculate the area.


## Example 1: Volume of a Shell



Top View


The insulated box shown is made from a large cube with a hollow inside that is a right rectangular prism with a square base. The figure at right is what the box looks like from above.
a. Calculate the volume of the outer box.
b. Calculate the volume of the inner prism.
c. Describe in words how you would find the volume of the insulation.
d. Calculate the volume of the insulation in cubic centimeters.
e. Calculate the amount of water the box can hold in liters.

## Exercise 1: Designing a Brick Planter (10 minutes)

You have been asked by your school to design a brick planter that will be used by classes to plant flowers. The planter will be built in the shape of a right rectangular prism with no bottom so water and roots can access the ground beneath.
The exterior dimensions are to be $12 \mathrm{ft} . \times 9 \mathrm{ft} . \times 2 \frac{1}{2} \mathrm{ft}$. The bricks used to construct the planter have are 6 inches long, $3 \frac{1}{2}$ inches wide, and 2 inches high.
a. What are the interior dimensions of the planter if the thickness of the planter's walls is equal to the length of the bricks?
b. What is the volume all the brick that forms the planter?
c. If you are going to fill the planter $\frac{3}{4}$ full of soil, how much soil will you need to purchase, and what will be the height of the soil?
d. How many bricks are needed to construct the planter?
e. The bricks used in this project cost $\$ 0.82$ each and weigh 4.5 lb . each. The supply company charges a delivery fee of $\$ 15$ per whole ton ( 2000 lbs ) over 4000 pounds. How much will your school pay for the bricks (including delivery) to construct the planter?
f. A cubic foot of top soil weighs between 75 and 100 lb . How much will the soil in the planter weigh?
g. If the topsoil costs $\$ 0.88$ per each cubic foot, calculate the total cost of materials that will be used to construct the planter.

## Exercise 2: Design a Feeder

You did such a good job designing the planter that a local farmer has asked you to design a feeder for the animals on his farm. Your feeder will hold at least 600 cubic centimeters, but not more than 800 cubic centimeters of grain when it is full. It will sit on the ground (don't worry about the base or what holds it up in your design). It will be built of metal that is $2 \frac{1}{4} \mathrm{~cm}$ thick and must be a prism, but not a rectangular prism. Sketch your design below with dimensions. Calculate the volume of grain that it can hold and the amount of metal needed to construct the feeder.

The farmer needs a cost estimate. Calculate the cost of constructing and filling the feeder if grain sells for $\$ 0.40 / \mathrm{cm}^{3}$ and metal sells for $\$ 7.50 /$ square meter.

## Problem Set

1. A child's toy is constructed by cutting a right triangular prism out of a right rectangular prism.

a. Calculate the volume of the rectangular prism.
b. Calculate the volume of the triangular prism.
c. Calculate the volume of the material remaining in the rectangular prism.
d. What is the largest number of triangular prisms that can be cut from the rectangular prism?
e. What is the surface area of the triangular prism (assume there is no top or bottom)?
2. A landscape designer is constructing a flower bed in the shape of a right trapezoidal prism. He needs to run three identical square prisms through the bed for drainage.

a. What is the volume of the bed without the drainage pipes?
b. What is the total volume of the three drainage pipes?
c. What is the volume of soil that can fit in the bed once the pipes are in place, assuming the amount of soil is filled to $3 / 4$ of the height of the planter?
d. What is the height of the soil?
e. If the bed is made of $8 \mathrm{ft} \times 4 \mathrm{ft}$. pieces of plywood, how many pieces of plywood will the landscape designer need to construct the bed without the drainage pipes?
f. If the plywood needed to construct the bed costs $\$ 35$ per $8 \mathrm{ft} . \times 4 \mathrm{ft}$. piece, the drainage pipes cost $\$ 125$ each, and the soil costs $\$ 1.25 /$ cubic foot, how much does it cost to construct and fill the bed?
