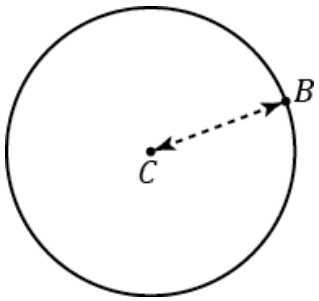
Lesson 16: The Most Famous Ratio of All

Classwork

Opening Exercise

a. Using a compass, draw a circle like the picture to the right.



C is the *center* of the circle.
The distance between C and B is the *radius* of the circle.

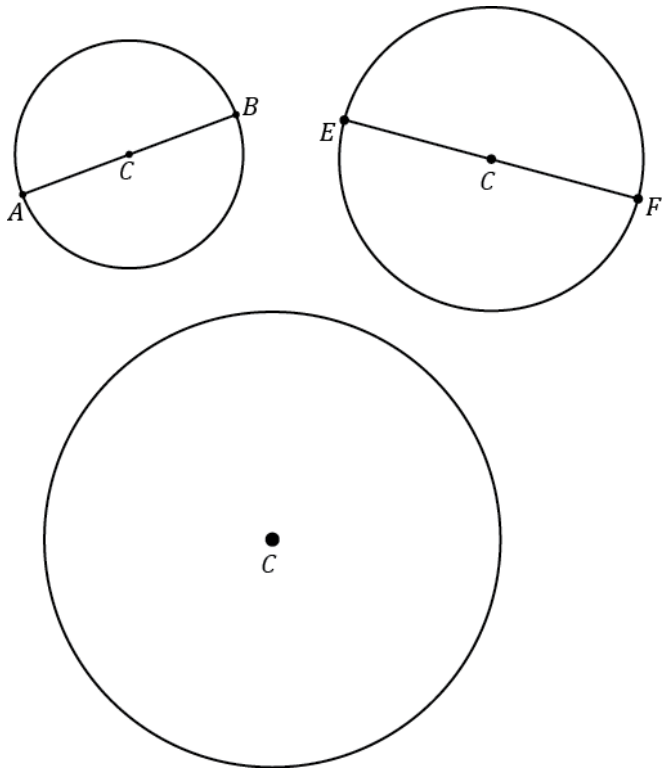
b. Write your own definition for the term circle.

c. Extend segment CB to a segment AB in part (a), where A is also a point on the circle.

The length of the segment AB is called the *diameter of the circle.*

d. The diameter is _____ as long as radius.

e. Measure the radius and diameter of each circle. The center of each circle is labeled C.



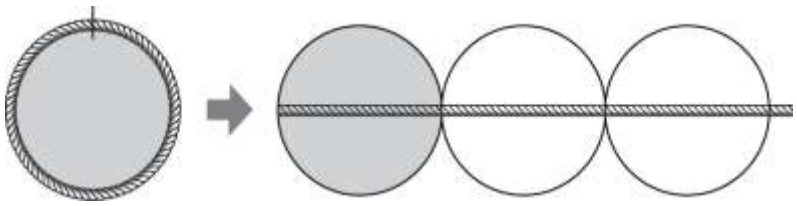
f. Draw a circle of radius 6 cm.

Example 1

The ratio of the circumference to its diameter is always the same for any circle. The value of this ratio,

$$\frac{Circumference}{Diameter}$$

is called the number *pi* and is represented by the symbol π.

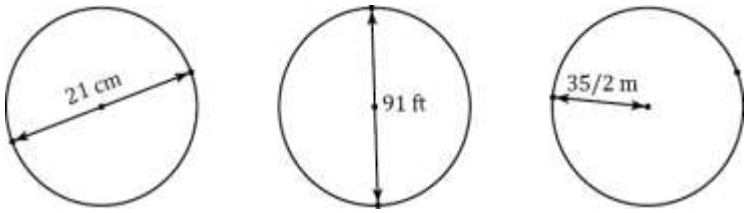


Since the circumference is a little greater than 3 times the diameter, π is a number that is a little greater than 3. State: Use the symbol π to represent this special number. Pi is a non-terminating, non-repeating decimal and mathematicians use the symbol π or approximate representations as more convenient ways to represent pi.

- $\pi \approx 3.14$ or $\frac{22}{7}$.
- The ratios of circumference : diameter and $\pi : 1$ are equal.
- Circumference of a Circle $= \pi \times$ Diameter.

Exercise 2

a. The following circles are not drawn to scale. Find the circumference of each circle. (Use $\frac{22}{7}$ as an approximation for π.)

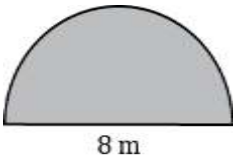


b. The radius of a paper plate is 11.7 cm. Find the circumference to the nearest tenth. (Use 3.14 as an approximation for π.)

c. The radius of a paper plate is 11.7 cm. Find the circumference to the nearest hundredth. (Use the π button on your calculator as an approximation for π.)

d. A circle has a radius of r cm and a circumference of C cm. Write a formula that expresses the value of C in terms of r and π.

e. The figure below is in the shape of a semicircle. A semicircle is an arc that is "half" of a circle. Find the perimeter of the shape. (Use 3.14 for π.)



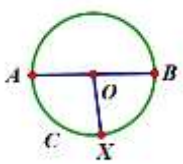
Relevant Vocabulary

Circle: Given a point C in the plane and a number $r > 0$, the *circle with center C and radius r* is the set of all points in the plane that are distance r from the point C.

Radius of a circle: The *radius* is the length of any segment whose endpoints are the center of a circle and a point that lies on the circle.

Diameter of a circle: The *diameter of a circle* is the length of any segment that passes through the center of a circle whose endpoints lie on the circle. If r is the *radius* of a circle, then the diameter is $2r$.

The word *diameter* can also mean the segment itself. Context determines how the term is being used: "the diameter" usually refers to the length of the segment, while "a diameter" usually refers to a segment. Similarly, "a radius" can refer to a segment from the center of a circle to a point on the circle.

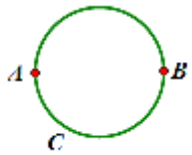


Circle C

Radii: $\overline{OA}, \overline{OB}, \overline{OX}$

Diameter: $\overline{AB}$

Circumference

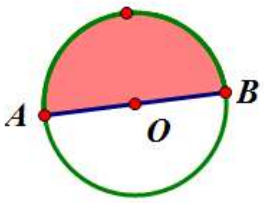


Circumference: The circumference of a circle is the distance around a circle.

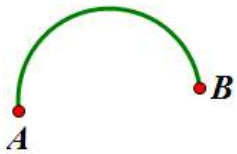
Pi: The number *pi*, denoted by π, is the value of the ratio given by the circumference to the diameter, that is $\boldsymbol{\pi} = \frac{circumference}{diameter}$. The most commonly used approximations for π is 3.14 or $\frac{22}{7}$.

Semicircle: Let C be a circle with center O, and let A and B be the endpoints of a diameter. A *semicircle* is the set containing A, B, and all points that lie in a given half-plane determined by AB (diameter) that lie on circle C.

Half-plane

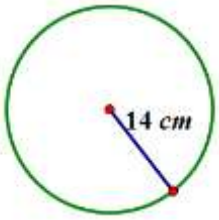


Semi-circle

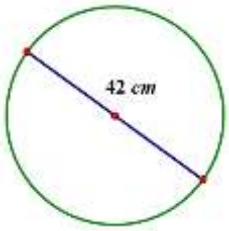


Problem Set

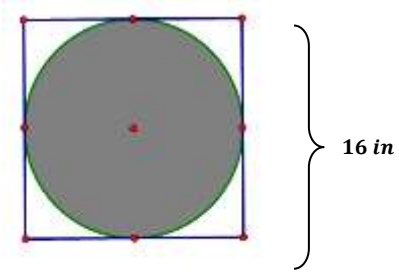
1. Find the circumference.
   a. Give an exact answer in terms of π.
   b. Use $\pi \approx \frac{22}{7}$ and express your answer as a fraction in lowest terms.
   c. Use $the\ \pi$ button on your calculator and express your answer to the nearest hundredth.



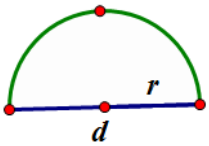
2. Find the circumference.
   a. Give an exact answer in terms of π.
   b. Use $\pi \approx \frac{22}{7}$ and express your answer as a fraction in lowest terms.



3. The figure shows a circle within a square. Find the circumference of the circle. Let $\pi \approx 3.14$.

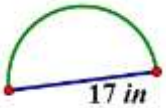


4. Consider the diagram of a semicircle shown.



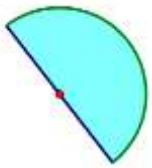
   a. Explain in words how to determine the perimeter of a semicircle.
   b. Using "d" to represent the diameter of the circle, write an algebraic expression that will result in the perimeter of a semicircle.
   c. Write another algebraic expression to represent the perimeter of a semicircle using r to represent the radius of a semicircle.

5. Find the perimeter of the semicircle. Let $\pi \approx 3.14$.

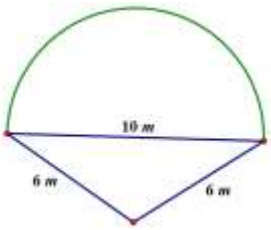


6. Ken's landscape gardening business makes odd shaped lawns which include semicircles. Find the length of the edging material needed to border the two lawn designs. Use 3.14 for π.

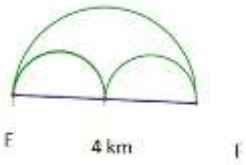
   a. The radius of this flower bed is 2.5 m.



   b. The diameter of the semicircular section is 10 m, and the lengths of the sides of the two sides are 6 m.



7. Mary and Margaret are looking at a map of a running path in a local park. Which is the shorter path from E to F: along the two semicircles or along the larger semicircle? If one path is shorter, how much shorter is it?



8. Alex the electrician needs 34 yards of electrical wire to complete a job. He has a coil of wiring in his workshop. The coiled up wire is 18 inches in diameter and is made up of 21 circles of wire. Will this coil be enough to complete the job?