

## Lesson 6: Collecting Rational Number Like Terms

### Classwork

#### Opening Exercise

Do the computations, leaving your answers in simplest/standard form. Show your steps.

1. Terry weighs 40 kg. Janice weighs  $2\frac{3}{4}$  kg less than Terry. What is their combined weight?

2.  $2\frac{2}{3} - 1\frac{1}{2} - \frac{4}{5}$

3.  $\frac{1}{5} + (-4)$

4.  $4\left(\frac{3}{5}\right)$

5. Mr. Jackson bought  $1\frac{3}{5}$  lbs of beef. He cooked  $\frac{3}{4}$  of it for lunch. How much does he have left?

6.  $\frac{2}{3}n - \frac{3}{4}n + \frac{1}{6}n + 2\frac{2}{9}n$

### Example 1

Rewrite the expression in standard form by collecting like terms.

$$\frac{1}{2}a + 2\frac{2}{3}b + \frac{1}{5} - \frac{1}{4}a - 1\frac{1}{2}b + \frac{3}{5} + \frac{3}{4}a - 4 - \frac{4}{5}b$$

$$\frac{1}{2}a + 2\frac{2}{3}b + \frac{1}{5} + \left(-\frac{1}{4}a\right) + \left(-1\frac{1}{2}b\right) + \frac{3}{5} + \frac{3}{4}a + (-4) + \left(-\frac{4}{5}b\right)$$

Subtraction as adding the inverse

$$\frac{1}{2}a + \left(-\frac{1}{4}a\right) + \frac{3}{4}a + 2\frac{2}{3}b + \left(-1\frac{1}{2}b\right) + \left(-\frac{4}{5}b\right) + \frac{1}{5} + \frac{3}{5} + (-4)$$

Any order property (commutative property)

$$\left(\frac{1}{2} + \left(-\frac{1}{4}\right) + \frac{3}{4}\right)a + \left(2\frac{2}{3} + \left(-1\frac{1}{2}\right) + \left(-\frac{4}{5}\right)\right)b + \left(\frac{4}{5} + (-4)\right)$$

Collecting like terms by applying distributive property

$$a + \frac{11}{30}b - \frac{16}{5}$$

Arithmetic rules for rational numbers

The expression with eight terms can be rewritten with a minimum of three terms.

### Exercise 1

For the following exercises, predict how many terms the resulting expression will have after collecting like terms. Then, write the expression in standard form by collecting like terms.

a.  $\frac{2}{5}g - \frac{1}{6} - g + \frac{3}{10}g - \frac{4}{5}$

b.  $i + 6i - \frac{3}{7}i + \frac{1}{3}h + \frac{1}{2}i - h + \frac{1}{4}h$

**Example 2**

At a store, a shirt was marked down in price by ten dollars. A pair of pants doubled in price. Following these changes, the price of every item in the store was cut in half. Write two different expressions that represent the new cost of the items, using  $s$  for the cost of each shirt and  $p$  for the cost of a pair of pants. Explain the different information each one shows.

**Exercise 2**

Continuing with Example 2, write two different expressions that represent the total cost of the items if tax was  $\frac{1}{10}$  of the original price. Explain the different information each shows.

### Example 3

As a class, write each expression in standard form by collecting like terms. Justify each step.

$$5\frac{1}{3} - \left(3\frac{1}{3}\right)\left(\frac{1}{2}x - \frac{1}{4}\right)$$

$$\frac{16}{3} + \left(-\frac{10}{3}\right)\left(\frac{1}{2}x\right) + \left(-\frac{10}{3}\right)\left(-\frac{1}{4}\right)$$

Write mixed numbers as improper fractions, then distribute.

$$\frac{16}{3} + \left(-\frac{5}{3}x\right) + \frac{5}{6}$$

Any grouping (associative) and arithmetic rules for multiplying rational numbers

$$-\frac{5}{3}x + \left(\frac{32}{6} + \frac{5}{6}\right)$$

Commutative property and associative property of addition, collect like terms

$$-\frac{5}{3}x + \frac{37}{6}$$

Apply arithmetic rule for adding rational numbers

### Exercise 3

Rewrite the following expressions in standard form by finding the product and collecting like terms.

a.  $-6\frac{1}{3} - \frac{1}{2}\left(\frac{1}{2} + y\right)$

b.  $\frac{2}{3} + \frac{1}{3}\left(\frac{1}{4}f - 1\frac{1}{3}\right)$

**Example 4**

Model how to write the expression in standard form using rules of rational numbers.

$$\frac{x}{20} + \frac{2x}{5} + \frac{x+1}{2} + \frac{3x-1}{10}$$

$\begin{array}{r} \frac{x}{20} + \frac{4(2x)}{20} + \frac{10(x+1)}{20} + \frac{2(3x-1)}{20} \\ \hline \frac{x + 8x + 10x + 10 + 6x - 2}{20} \\ \hline \frac{25x + 8}{20} \\ \hline \frac{5}{4}x + \frac{2}{5} \end{array}$	$\begin{array}{r} \frac{1}{20}x + \frac{2}{5}x + \frac{1}{2}x + \frac{1}{2} + \frac{3}{10}x - \frac{1}{10} \\ \left(\frac{1}{20} + \frac{2}{5} + \frac{1}{2} + \frac{3}{10}\right)x + \left(\frac{1}{2} - \frac{1}{10}\right) \\ \left(\frac{1}{20} + \frac{8}{20} + \frac{10}{20} + \frac{6}{20}\right)x + \left(\frac{5}{10} - \frac{1}{10}\right) \\ \hline \frac{5}{4}x + \frac{2}{5} \end{array}$
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Evaluate the original expression and their answers when  $x = 20$ . Do you get the same number?

$\begin{array}{r} \frac{x}{20} + \frac{2x}{5} + \frac{x+1}{2} + \frac{3x-1}{10} \\ \frac{20}{20} + \frac{2(20)}{5} + \frac{20+1}{2} + \frac{3(20)-1}{10} \\ 1 + 8 + \frac{21}{2} + \frac{59}{10} \\ 9 + \frac{105}{10} + \frac{59}{10} \\ 9 + \frac{164}{10} \\ 9 + 16\frac{4}{10} \\ 25\frac{2}{5} \end{array}$	$\begin{array}{r} \frac{5}{4}x + \frac{2}{5} \\ \frac{5}{4}(20) + \frac{2}{5} \\ 25 + \frac{2}{5} \\ 25\frac{2}{5} \end{array}$
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**Exercise 4**

Rewrite the following expression in standard form by finding common denominators and collecting like terms.

$$\frac{2h}{3} - \frac{h}{9} + \frac{h-4}{6}$$

Example 5

Rewrite the following expression in standard form.

$$\frac{2(3x - 4)}{6} - \frac{5x + 2}{8}$$

Method 1:

$$\begin{aligned} & \frac{1(3x - 4)}{3} - \frac{5x + 2}{8} \\ & \frac{8(3x - 4)}{24} - \frac{3(5x + 2)}{24} \\ & \frac{((24x - 32) - (15x + 6))}{24} \\ & \frac{(24x - 32 - 15x - 6)}{24} \\ & \frac{9x - 38}{24} \\ & \frac{9x}{24} - \frac{38}{24} \\ & \frac{3}{8}x - \frac{19}{12} \end{aligned}$$

Method 2a:

$$\begin{aligned} & \frac{6x - 8}{6} - \frac{5x + 2}{8} \\ & \frac{4(6x - 8)}{24} - \frac{3(5x + 2)}{8} \\ & \frac{(24x - 32 - 15x - 6)}{24} \\ & \frac{9x - 38}{24} \\ & \frac{9x}{24} - \frac{38}{24} \\ & \frac{3}{8}x - \frac{19}{12} \end{aligned}$$

Method 2b:

$$\begin{aligned} & \frac{6}{6}x - \frac{8}{6} - \frac{5}{8}x - \frac{2}{8} \\ & x - \frac{4}{3} - \frac{5x}{8} - \frac{1}{4} \\ & 1x - \frac{5}{8}x - \frac{4}{3} - \frac{1}{4} \\ & \frac{3}{8}x - \frac{16}{12} - \frac{3}{12} \\ & \frac{3}{8}x - \frac{19}{12} \end{aligned}$$

Method 3:

$$\begin{aligned} & \frac{1}{3}(3x - 4) - \left(\frac{5x}{8} + \frac{1}{4}\right) \\ & x - \frac{4}{3} - \frac{5}{8}x - \frac{1}{4} \\ & 1x - \frac{5}{8}x - \frac{4}{3} - \frac{1}{4} \\ & \frac{3}{8}x - \frac{16}{12} - \frac{3}{12} \\ & \frac{3}{8}x - \frac{19}{12} \end{aligned}$$

Exercise 5

Write the following expression in standard form.

$$\frac{2x - 11}{4} - \frac{3(x - 2)}{10}$$

### Problem Set

- Write the indicated expressions.
  - $\frac{1}{2}m$  inches in feet.
  - The perimeter of a square with  $\frac{2}{3}g$  cm sides.
  - The number of pounds in 9 oz.
  - The average speed of a train that travels  $x$  miles in  $\frac{3}{4}$  hour.
  - Devin is  $1\frac{1}{4}$  years younger than Eli. April is  $\frac{1}{5}$  as old as Devin. Jill is 5 years older than April. If Eli is  $E$  years old, what is Jill's age in terms of  $E$ ?
- Rewrite the expressions by collecting like terms.
  - $\frac{1}{2}k - \frac{3}{8}k$
  - $\frac{2r}{5} + \frac{7r}{15}$
  - $-\frac{1}{3}a - \frac{1}{2}b - \frac{3}{4} + \frac{1}{2}b - \frac{2}{3}b + \frac{5}{6}a$
  - $-p + \frac{3}{5}q - \frac{1}{10}q + \frac{1}{9} - \frac{1}{9}p + 2\frac{1}{3}p$
  - $\frac{5}{7}y - \frac{y}{14}$
  - $\frac{3n}{8} - \frac{n}{4} + 2\frac{n}{2}$
- Rewrite the expressions by using the distributive property and collecting like terms.
  - $\frac{4}{5}(15x - 5)$
  - $\frac{4}{5}(\frac{1}{4}c - 5)$
  - $2\frac{4}{5}v - \frac{2}{3}(4v + 1\frac{1}{6})$
  - $8 - 4(\frac{1}{8}r - 3\frac{1}{2})$
  - $\frac{1}{7}(14x + 7) - 5$
  - $\frac{1}{5}(5x - 15) - 2x$
  - $\frac{1}{4}(p + 4) + \frac{3}{5}(p - 1)$
  - $\frac{7}{8}(w + 1) + \frac{5}{6}(w - 3)$
  - $\frac{4}{5}(c - 1) - \frac{1}{8}(2c + 1)$
  - $\frac{2}{3}(h + \frac{3}{4}) - \frac{1}{3}(h + \frac{3}{4})$
  - $\frac{2}{3}(h + \frac{3}{4}) - \frac{2}{3}(h - \frac{3}{4})$
  - $\frac{2}{3}(h + \frac{3}{4}) + \frac{2}{3}(h - \frac{3}{4})$
  - $\frac{k}{2} - \frac{4k}{5} - 3$
  - $\frac{3t+2}{7} + \frac{t-4}{14}$
  - $\frac{9x-4}{10} + \frac{3x+2}{5}$
  - $\frac{3(5g-1)}{4} - \frac{2g+7}{6}$
  - $-\frac{3d+1}{5} + \frac{d-5}{2} + \frac{7}{10}$
  - $\frac{9w}{6} + \frac{2w-7}{3} - \frac{w-5}{4}$
  - $\frac{1+f}{5} - \frac{1+f}{3} + \frac{3-f}{6}$

