Lesson 1 MATHEMATICS CURRICULUM

Lesson 1: Generating Equivalent Expressions

Classwork

Each envelope contains a number of triangles and a number of quadrilaterals. For this exercise, let t represent the number of triangles, and let q represent the number of quadrilaterals.	
b.	You and your partner have the same number of triangles and quadrilaterals in your envelopes. Write an expression that represents the total number of sides that you and your partner have. If possible, write more than one expression to represent this total.
c.	Each envelope in the class contains the same number of triangles and quadrilaterals. Write an expression that represents the total number of sides in the room.
٩	Use the given values of t and a and your expression from part (a) to determine the number of sides that should be

Use the given values of t and q, and your expression from part (a), to determine the number of sides that should be found in your envelope.

e. Use the same values for t and q, and your expression from part (b), to determine the number of sides that should be contained in your envelope and your partner's envelope combined.

f. Use the same values for t and q, and your expression from part (c), to determine the number of sides that should be contained in all of the envelopes combined.

g. What do you notice about the various expressions in parts (e) and (f)?

Example 1: Any Order, Any Grouping Property with Addition

- a. Rewrite 5x + 3x and 5x 3x by combining like terms.
 - Write the original expressions and expand each term using addition. What are the new expressions equivalent to?

b. Find the sum of 2x + 1 and 5x.

c. Find the sum of -3a + 2 and 5a - 3.

Example 2: Any Order, Any Grouping with Multiplication

Find the product of 2x and 3.

Example 3: Any Order, Any Grouping in Expressions with Addition and Multiplication

- a. 3(2x)
- b. 4y(5)
- c. $4 \cdot 2 \cdot z$
- d. 3(2x) + 4y(5)
- e. $3(2x) + 4y(5) + 4 \cdot 2 \cdot z$
- f. Alexander says that 3x + 4y is equivalent to (3)(4) + xy because of any order, any grouping. Is he correct? Why or why not?

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Relevant Vocabulary

<u>Variable (description)</u>: A *variable* is a symbol (such as a letter) that represents a number, i.e., it is a placeholder for a number.

<u>Numerical Expression (description)</u>: A *numerical expression* is a number, or it is any combination of sums, differences, products, or divisions of numbers that evaluates to a number.

<u>Value of a Numerical Expression</u>: The value of a numerical expression is the number found by evaluating the expression.

Expression (description): An *expression* is a numerical expression, or it is the result of replacing some (or all) of the numbers in a numerical expression with variables.

Equivalent Expressions: Two expressions are *equivalent* if both expressions evaluate to the same number for every substitution of numbers into all the letters in both expressions.

An expression in Expanded Form: An expression that is written as sums (and/or differences) of products whose factors are numbers, variables, or variables raised to whole number powers is said to be in *expanded form*. A single number, variable, or a single product of numbers and/or variables is also considered to be in expanded form. Examples of expressions in expanded form include: 324, 3x, 5x + 3 - 40, x + 2x + 3x, etc.

<u>Term (description)</u>: Each summand of an expression in expanded form is called a *term*. For example, the expression 2x + 3x + 5 consists of 3 terms: 2x, 3x, and 5.

<u>Coefficient of the Term (description)</u>: The number found by multiplying just the numbers in a term together. For example, given the product $2 \cdot x \cdot 4$, its equivalent term is 8x. The number 8 is called the coefficient of the term 8x.

<u>An Expression in Standard Form</u>: An expression in expanded form with all its like terms collected is said to be in *standard form*. For example, 2x + 3x + 5 is an expression written in expanded form; however, to be written in standard form, the like terms 2x and 3x must be combined. The equivalent expression 5x + 5 is written in standard form.

Lesson Summary

Terms that contain exactly the same variable symbol can be combined by addition or subtraction because the variable represents the same number. Any order, any grouping can be used where terms are added (or subtracted) in order to group together like terms. Changing the orders of the terms in a sum does not affect the value of the expression for given values of the variable(s).

Problem Set

For problems 1–9, write equivalent expressions by combining like terms. Verify the equivalence of your expression and the given expression by evaluating each for the given values: a = 2, b = 5, and c = -3.

1.
$$3a + 5a$$

2.
$$8b - 4b$$

3.
$$5c + 4c + c$$

4.
$$3a + 6 + 5$$

5.
$$8b + 8 - 4b$$

6.
$$5c - 4c + c$$

7.
$$3a + 6 + 5a - 2$$

8.
$$8b + 8 - 4b - 3$$

9.
$$5c - 4c + c - 3c$$

Use any order, any grouping to write equivalent expressions by combining like terms. Then verify the equivalence of your expression to the given expression by evaluating for the value(s) given in each problem.

10.
$$3(6a)$$
; for $a = 3$

11.
$$5d(4)$$
; for $d = -2$

12.
$$(5r)(-2)$$
; for $r = -3$

13.
$$3b(8) + (-2)(7c)$$
; for $b = 2, c = 3$

14.
$$-4(3s) + 2(-t)$$
; for $s = \frac{1}{2}$, $t = -3$

15.
$$9(4p) - 2(3q) + p$$
; for $p = -1, q = 4$

16.
$$7(4g) + 3(5h) + 2(-3g)$$
; $g = \frac{1}{2}$, $h = \frac{1}{3}$

The problems below are follow-up questions to Example 1b from Classwork: Find the sum of 2x + 1 and 5x.

- 17. Jack got the expression 7x + 1, then wrote his answer as 1 + 7x. Is his answer an equivalent expression? How do you know?
- 18. Jill also got the expression 7x + 1, then wrote her answer as 1x + 7. Is her expression an equivalent expression? How do you know?