LESSON 20

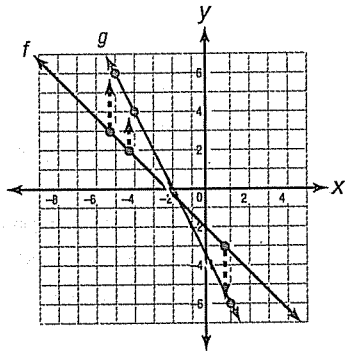
Stretching and Shrinking Functions

Vertical Stretches and Shrinks

UNDERSTAND Translations and reflections do not change the size of the graph being transformed, but stretches and shrinks do. After a stretch or a shrink, the new graph looks wider or narrower than the original graph.

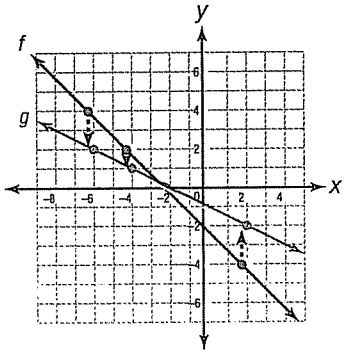
Multiplying the output of a function, $f(x)$, by a constant stretches or shrinks the graph in the vertical direction.

For $g(x) = kf(x)$ where $|k| > 1$, the graph of g is a **vertical stretch** of the graph of f. A vertical stretch pulls the points on the graph away from the x-axis.



The graphs of f and g above illustrate the vertical stretch $g(x) = 2f(x)$. Notice how each point on g is twice as far from the x-axis as its corresponding point on f.

For $g(x) = kf(x)$ where $0 < |k| < 1$, the graph of g is a **vertical shrink** of the graph of f. A vertical shrink pushes the points of a graph toward the x-axis.



The graphs of f and g above illustrate the vertical shrink $g(x) = \frac{1}{2}f(x)$. Notice how each point on g is half as far from the x-axis as its corresponding point on f.

Notice that, after a vertical stretch or shrink, the new function always has the same x-intercept as the original function. This point cannot be shrunk toward or stretched away from the x-axis because it is on the x-axis.

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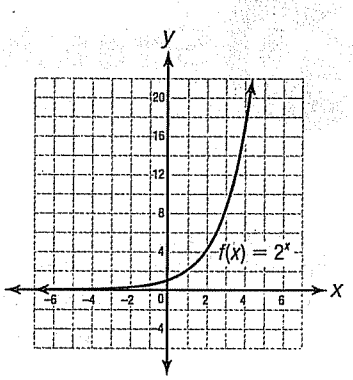
Connect

The exponential function $f(x) = 2^x$ is graphed on the right. Graph $g(x) = 4(2^x)$ on the same coordinate plane. Is g the result of a vertical stretch or a vertical shrink of f? by what factor?

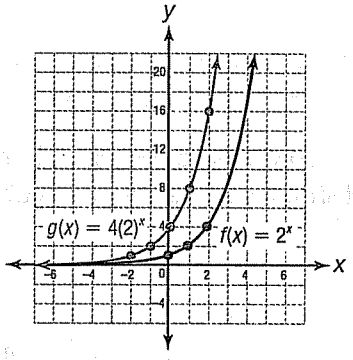


1 Create a table of values for g.

x	$g(x) = 4(2^x)$	$g(x)$
-2	$g(-2) = 4(2^{-2}) = 4\left(\frac{1}{4}\right) = 1$	1
-1	$g(-1) = 4(2^{-1}) = 4\left(\frac{1}{2}\right) = 2$	2
0	$g(0) = 4(2^0) = 4(1) = 4$	4
1	$g(1) = 4(2^1) = 4(2) = 8$	8
2	$g(2) = 4(2^2) = 4(4) = 16$	16

2 Graph g.

Plot those ordered pairs $(x, g(x))$ on the coordinate plane and connect them with a smooth curve.



3 Compare the graphs and identify the transformation.

The point (0, 1) on the graph of f is transformed to (0, 4) on the graph of g.

The point (1, 2) is transformed to (1, 8).

The point (2, 4) is transformed to (2, 16).

Notice that for each pair of points, the input is the same, but the output is 4 times as great.

The equation of g also shows that the output, $f(x)$, was multiplied by 4.

▶ The graph of g is the result of a vertical stretch of f by a factor of 4.

TRY

On the coordinate plane above, graph $h(x) = \frac{1}{4}(2^x)$. Is this a vertical stretch or a vertical shrink? by what factor?

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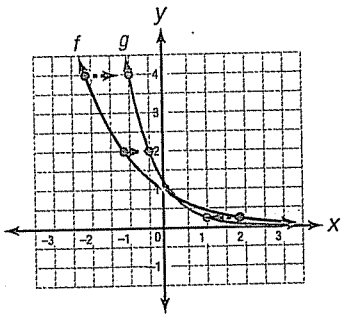
Horizontal Stretches and Shrinks

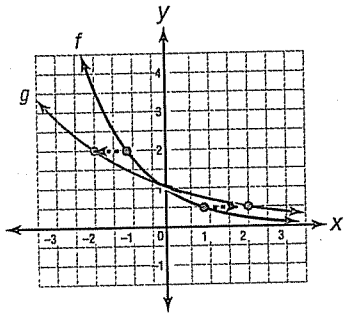
UNDERSTAND Multiplying the input of a function, x, by a constant stretches or shrinks the graph in the horizontal direction.

For $g(x) = f(kx)$ where $|k| > 1$, the graph of g is a **horizontal shrink** of the graph of f. A horizontal shrink pushes the points of a graph toward the y-axis.



The graphs of f and g above illustrate the horizontal shrink $g(x) = f(2x)$. Notice how each point on g is half as far from the y-axis as its corresponding point on f.

For $g(x) = f(kx)$ where $0 < |k| < 1$, the graph of g is a **horizontal stretch** of the graph of f. A horizontal stretch pulls the points of a graph away from the y-axis.



The graphs of f and g above illustrate the horizontal stretch $g(x) = f\left(\frac{1}{2}x\right)$. Notice how each point on g is twice as far from the y-axis as its corresponding point on f.

Notice that the factor of the stretch or shrink is the reciprocal of the constant. In the case of a horizontal stretch or shrink, k is not the factor by which a graph is stretched or shrunk; $\frac{1}{k}$ is.

Notice that, after a horizontal stretch or shrink, the new function always has the same y-intercept as the original function. This point cannot be shrunk toward or stretched away from the y-axis because it is on the y-axis.

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Connect

Graph the linear function $f(x) = \frac{1}{2}x + 2$. Then graph its image, g, resulting from the transformation described below.

$$g(x) = f(3x)$$

Is this an example of a horizontal stretch or shrink? by what factor?

1 Create a table of values for f.

x	$f(x) = \frac{1}{2}x + 2$	$f(x)$
-6	$f(-6) = \frac{1}{2}(-6) + 2$ $= -3 + 2 = -1$	-1
0	$f(0) = \frac{1}{2}(0) + 2 = 0 + 2 = 2$	2
6	$f(6) = \frac{1}{2}(6) + 2 = 3 + 2 = 5$	5

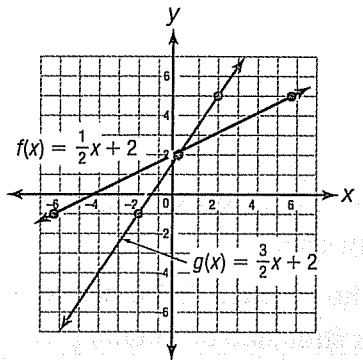
2 Write an equation and create a table of values for g.

Since $g(x) = f(3x)$, replace x with $3x$ to find the equation for g.

$g(x) = f(3x) = \frac{1}{2}(3x) + 2 = \frac{3}{2}x + 2.$

x	$g(x) = \frac{3}{2}x + 2$	$g(x)$
-2	$g(-2) = \frac{3}{2}(-2) + 2$ $= -3 + 2 = -1$	-1
0	$g(0) = \frac{3}{2}(0) + 2 = 0 + 2 = 2$	2
2	$g(2) = \frac{3}{2}(2) + 2 = 3 + 2 = 5$	5

3 Use those points to graph the functions.



4 Identify the transformation and the factor.

The points on the graph of g are closer to the y-axis than their corresponding points on the graph of f. A horizontal shrink pushes a graph toward the y-axis.

For $g(x) = f(3x)$, $k = 3$. So, the factor is $\frac{1}{k}$, or $\frac{1}{3}$.

▶ The graph of g is the result of a horizontal shrink of f by a factor of $\frac{1}{3}$.

CHECK

Enter the equations for f and g into your graphing calculator, graph them, and call up tables of values for them. Do the graphs and tables match what appears on this page?

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EXAMPLE A Graph the linear functions $f(x) = -4x - 3$ and $g(x) = -2x - 3$.

Use words to describe the transformation, including if it is a horizontal stretch or a horizontal shrink, and by what factor. Then write an equation for $g(x)$ in terms of $f(x)$.

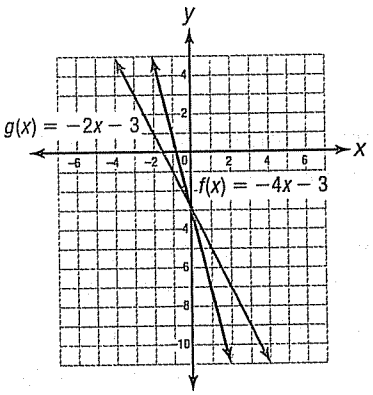
1 Use what you know about slope-intercept form to graph each line.

In $f(x) = -4x - 3$, the slope is -4 and the y-intercept is $(0, -3)$.

So, plot the point $(0, -3)$. Then count 4 units down and 1 unit to the right and plot a second point, $(1, -7)$. Draw and label the line for f.

In $g(x) = -2x - 3$, the slope is -2 and the y-intercept is $(0, -3)$.

So, plot the point $(0, -3)$. Then count 2 units down and 1 unit to the right and plot a second point, $(1, -5)$. Draw and label the line for g.



2 Identify the transformation.

The points on the graph of g are farther from the y-axis than their corresponding points on the graph of f. A horizontal stretch pulls a graph away from the y-axis.

3 Describe the transformation in words and as a function.

Compare the point $(-1, 1)$ on f to its corresponding point $(-2, 1)$ on g. The point on g is twice as far from the y-axis as the point on f.

▶ The transformation was a horizontal stretch by a factor of 2.

Since this transformation is horizontal, the stretch factor is equal to $\frac{1}{k}$, and, in function notation, k is multiplied by the input, x.

$2 = \frac{1}{k}$

$k = \frac{1}{2}$

▶ $g(x) = f\left(\frac{1}{2}x\right)$

DISCUSS

Does a stretch or shrink of the graph of a line always change its slope? Explain.

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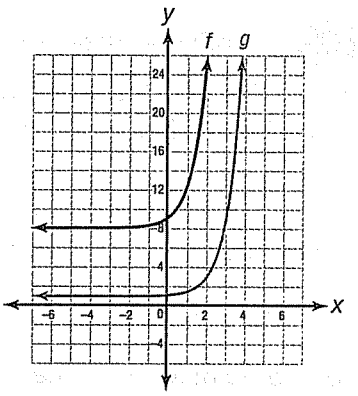
EXAMPLE B The graph of function f was transformed to create the graph of function g, as shown. Was f stretched or shrunk, horizontally or vertically, and by what factor? If $f(x) = 4^x + 8$, what is the equation of $g(x)$?



1 Examine the key features to determine how f was transformed.

As you move left on the graph of f, the values of $f(x)$ get very close to 8. So, f appears to have an asymptote at $y = 8$.

Many points on the graph of g lie below that asymptote, so points from the graph of f must have been moved much closer to the x-axis. Thus, g appears to be a vertical shrink of f.

2 Identify the factor of the shrink.

In a vertical shrink, the output is multiplied by a factor, k. So, compare points.

A vertical shrink has the form $g(x) = kf(x)$.

So, $k = \frac{g(x)}{f(x)}$ for all x.

The point (2, 24) on the graph of f was shrunk to (2, 3) on the graph of g.

$k = \frac{g(2)}{f(2)} = \frac{3}{24} = \frac{1}{8}$

3 Find the equation for $g(x)$.

$g(x) = \frac{1}{8}f(x) = \frac{1}{8}(4^x + 8) = \frac{1}{8} \cdot 4^x + 1$

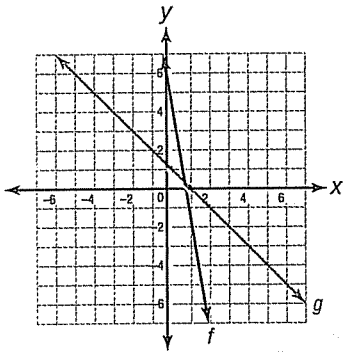
DISCUSS

Suppose $f(x) = 4^x + 8$ is shrunk horizontally by a factor of $\frac{1}{8}$ to form function h. Will the equation of h be $h(x) = 4^{\frac{1}{8}x} + 8$ or $h(x) = 4^{8x} + 8$? Explain.

Practice

Classify the graph of *g* as either a *vertical stretch* or a *vertical shrink* of the graph of *f*.

1.



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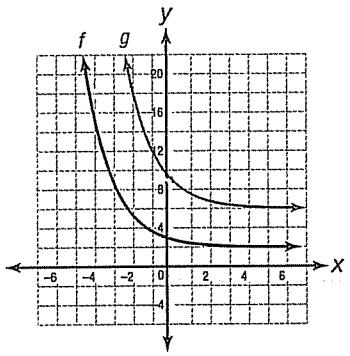
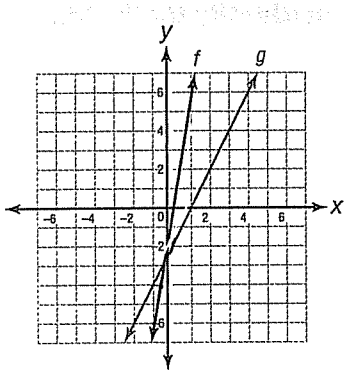


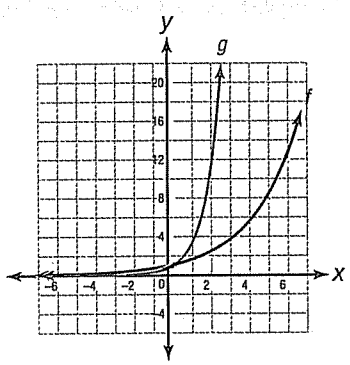
HINT: A vertical shrink draws points closer to the *x*-axis.

Classify the graph of *g* as either a *horizontal stretch* or a *horizontal shrink* of the graph of *f*.

3.



4.



HINT: Is *g* closer to or farther away from the *y*-axis?

Fill in each blank with an appropriate word, phrase, or expression.

5. A horizontal _____ pushes the points of a graph toward the *y*-axis.

6. A horizontal _____ pulls the points of a graph away from the *y*-axis.

7. If $g(x) = kf(x)$ and $|k| > 1$, then *g* is the result of a vertical stretch of *g* by a factor of _____.

8. If $g(x) = f(kx)$ and $|k| > 1$, then *g* is the result of a horizontal shrink of *g* by a factor of _____.

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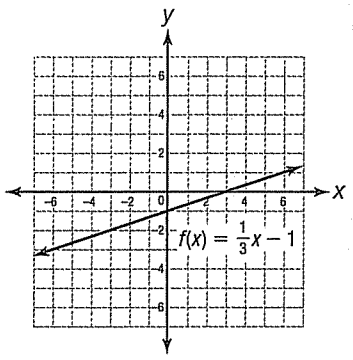
Graph each function *g* on the coordinate plane below it. Classify each graph of *g* as either a vertical stretch or a vertical shrink of the graph of *f*. Then identify the factor.

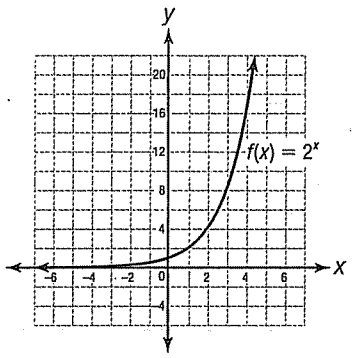
9. $g(x) = 2x - 6$



transformation: vertical _____

factor: _____

10. $g(x) = \frac{1}{2}(2^x)$

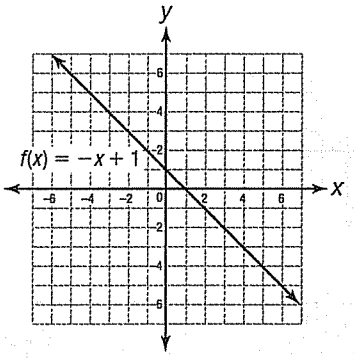


transformation: vertical _____

factor: _____

Graph each function *g* on the coordinate plane below it. Classify each graph of *g* as either a horizontal stretch or a horizontal shrink of the graph of *f*. Then identify the factor.

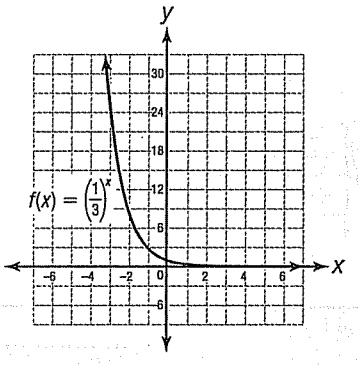
11. $g(x) = -5x + 1$



transformation: horizontal _____

factor: _____

12. $g(x) = \left(\frac{1}{3}\right)^{\frac{1}{2}x}$



transformation: horizontal _____

factor: _____

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Match each verbal description of a translation of $f(x) = 6^x$ with the equation of its transformed image by writing the correct letter next to each description.

13. vertical stretch by a factor of 8 _____

14. vertical shrink by a factor of $\frac{1}{8}$ _____

15. horizontal stretch by a factor of 8 _____

16. horizontal shrink by a factor of $\frac{1}{8}$ _____

A. $g(x) = 6^{8x}$

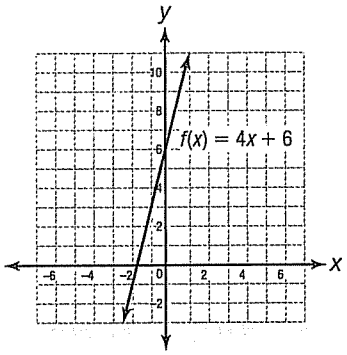
B. $g(x) = 8(6^x)$

C. $g(x) = 6^{\frac{1}{8}x}$

D. $g(x) = \frac{1}{8}(6^x)$

Graph each function g on the coordinate plane below it. Classify each graph of g as a stretch or a shrink of the graph of f, horizontal or vertical, and by what factor.

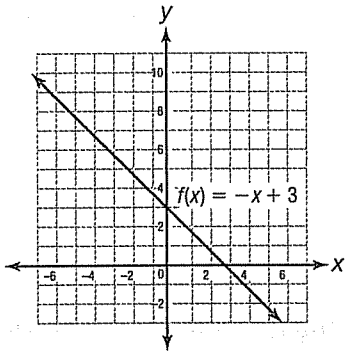
17. $g(x) = \frac{3}{2}f(x)$



transformation: _____

factor: _____

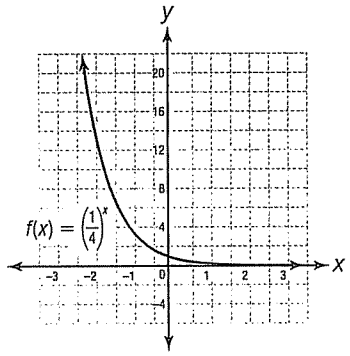
18. $g(x) = f\left(\frac{1}{3}x\right)$



transformation: _____

factor: _____

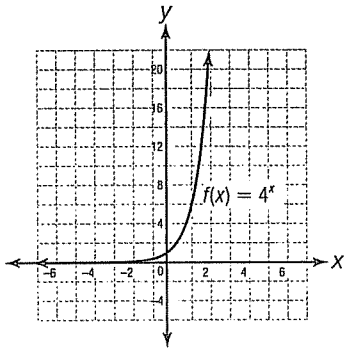
19. $g(x) = \frac{1}{2}f(x)$



transformation: _____

factor: _____

20. $g(x) = f(2x)$



transformation: _____

factor: _____

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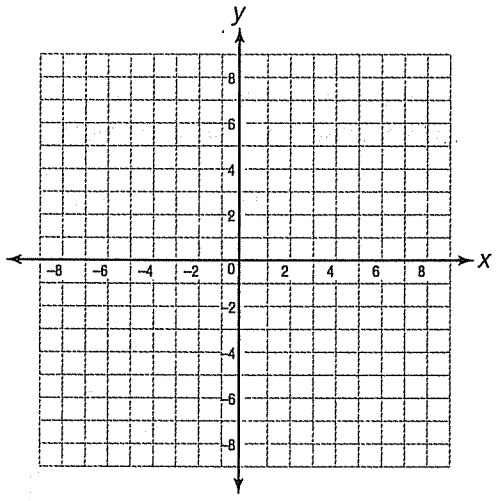
Graph the functions described and provide the desired information about them.

21. RESTATE Graph $f(x) = 2x - 2$ and its image, g, after a horizontal stretch by a factor of 4. Write equations for $g(x)$ in terms of $f(x)$ and in terms of x.

$g(x) =$ _____ $f($_____$x)$

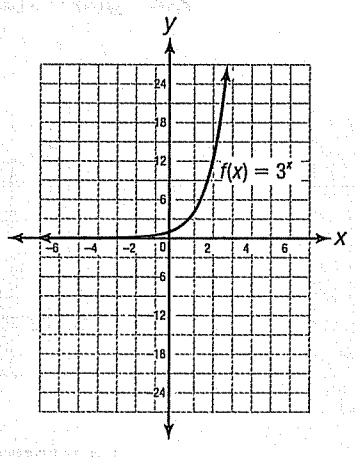
$g(x) =$ _____ $x -$ _____



22. DISTINGUISH The graph of $f(x) = 3^x$ is shown. Graph $g(x) = -\frac{1}{3}f(x)$. Then describe the two transformations needed to create the image, g, from the graph of f.

_____

_____



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