LESSON 15

7.EE.4.a

# Solving Word Problems Using Equations

## 1 GETTING THE IDEA

An **algebraic equation** is a number sentence with an equal sign that includes one or more **variables**. A variable is a symbol used to represent one or more numbers.

Solving an equation means finding the value or values of the variable that make the equation true. You can do this by using **inverse operations** to isolate the variable on one side of the equation. Inverse operations are operations that undo one another.

As long as you apply the same operations to both sides of the equation, the equation will still be true. These properties of equality will help you keep equations balanced.

For rational numbers $a$, $b$, and $c$:

| | |
|---|---|
| **Addition property of equality** | If $a = b$, then $a + c = b + c$. |
| **Subtraction property of equality** | If $a = b$, then $a - c = b - c$. |
| **Multiplication property of equality** | If $a = b$, then $ac = bc$. |
| **Division property of equality** | If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$. |

### Example 1

Solve for $x$: $-85x + 74 = 499$

**Strategy** Use inverse operations to isolate the variable.

**Step 1** Use inverse operations to get the variable term by itself.

74 is added to $-85x$, so use the inverse of addition.

$-85x + 74 = 499$ Subtract 74 from both sides.

$-85x + 74 - 74 = 499 - 74$

$-85x = 425$ The variable term, $-85x$, is by itself.

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**Step 2** Use the division property of equality to isolate the variable.

$-85$ is multiplied by $x$, so use the inverse of multiplication.

$-85x = 425$ Divide both sides by $-85$.

$$\frac{-85x}{-85} = \frac{425}{-85}$$

$$x = -5$$

**Step 3** Check your solution.

Verify that $x = -5$ makes the original equation true.

$$-85x + 74 = 499$$

$$-85(-5) + 74 \stackrel{?}{=} 499$$

$$425 + 74 \stackrel{?}{=} 499$$

$$499 = 499 \checkmark$$

**Solution** $x = -5$

Some real-world problems can be represented by an algebraic equation. You can write and solve an equation in order to solve them.

## Example 2

Molly had a $10 gift certificate to download music. After buying 6 songs that each cost the same amount, Molly has $4.06 left. How much did Molly pay for each song?

**Strategy** **Write and solve an equation.**

**Step 1** Use a verbal model to write an equation.

Let $c$ represent the cost of each song. The amount paid for the 6 songs plus the remaining balance on the card is equal to the original value of the certificate.

| (cost of 6 songs) | | (amount left) | | (original value) |
|---|---|---|---|---|
| $(6 \cdot c)$ | + | 4.06 | = | 10 |

The equation is $6c + 4.06 = 10$.

**Step 2** Use inverse operations to isolate x.

$6c + 4.06 = 10$ Subtract 4.06 from both sides.

$$6c + 4.06 - 4.06 = 10 - 4.06$$

$$6c = 5.94$$

$$\frac{6c}{6} = \frac{5.94}{6}$$ Divide both sides by 6.

$$c = 0.99$$

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**Step 3** Check your solution.

$$6c + 4.06 = 10$$
$$6(0.99) + 4.06 \stackrel{?}{=} 10$$
$$5.94 + 4.06 \stackrel{?}{=} 10$$
$$10 = 10 \checkmark$$

**Solution** **Molly paid $0.99 for each song.**

## Example 3

Solve for $x$: $\frac{2}{7}(x - 4) = 2$

**Strategy** **Use inverse operations to isolate the variable.**

**Step 1** Use inverse operations to get the variable term by itself.

To divide both sides by $\frac{2}{7}$, multiply by the reciprocal of $\frac{2}{7}$.

$\frac{2}{7}(x - 4) = 2$ Divide both sides by $\frac{2}{7}$.

$$\frac{2}{7} \div \frac{2}{7} \cdot (x - 4) = 2 \div \frac{2}{7}$$
$$\frac{2}{7} \cdot \frac{7}{2} \cdot (x - 4) = 2 \cdot \frac{7}{2}$$
$$x - 4 = 7$$

**Step 2** Use inverse operations to isolate the variable.

$x - 4 + 4 = 7 + 4$ Add 4 to both sides.

$x = 11$ Simplify.

**Step 3** Check your solution.

$$\frac{2}{7}(x - 4) = 2$$
$$\frac{2}{7}(11 - 4) \stackrel{?}{=} 2$$
$$\frac{2}{7}(7) \stackrel{?}{=} 2$$
$$2 = 2 \checkmark$$

**Solution** $x = 11$

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## Example 4

Three-fifths of a hotel's rooms are occupied. An additional 22 rooms are reserved. A total of 127 rooms are unavailable. How many rooms are in the hotel?

**Strategy** **Write and solve an equation.**

**Step 1** Use a verbal model to write an equation.

Let $r$ represent the total number of rooms in the hotel.

| (occupied rooms) | | (reserved rooms) | | (unavailable rooms) |
|---|---|---|---|---|
| $\left(\frac{3}{5} \cdot r\right)$ | + | 22 | = | 127 |

The equation $\frac{3}{5}r + 22 = 127$ can be used to find the value of $r$.

**Step 2** Use inverse operations to isolate the variable.

$\frac{3}{5}r + 22 = 127$ Subtract 22 from both sides.

$\frac{3}{5}r + 22 - 22 = 127 - 22$

$\frac{3}{5}r = 105$ Multiply both sides by $\frac{5}{3}$.

$\frac{5}{3}\left(\frac{3}{5}r\right) = \frac{5}{3}(105)$

$x = 175$

**Step 3** Check your solution.

$\frac{3}{5}(175) + 22 \stackrel{?}{=} 127$

$105 + 22 \stackrel{?}{=} 127$

$127 = 127$ ✓

**Solution** **The hotel has a total of 175 rooms.**

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## 2 COACHED EXAMPLE

**Jordan's school is having soil delivered for the school's vegetable garden. The soil costs $20 per cubic yard. There is also a $50 delivery fee. The total cost for soil and delivery is $200. How many cubic yards of soil did the school order?**

Let $x$ represent _____.

Use a verbal model to write an equation.

| (cost per cubic yard) | | (number of cubic yards) | | (delivery fee) | | (total cost) |
|---|---|---|---|---|---|---|
| _____ | · | $x$ | + | 50 | = | _____ |

Use inverse operations.

The variable term is added to 50, so use _____ to get the variable term by itself.

The number _____ is multiplied by $x$, so use _____ to isolate the variable.

Solve the equation. Fill in the blanks to show your steps.

(_____)$x$ + 50 = _____ Subtract _____ from both sides.

(_____)$x$ + 50 − _____ = _____ − _____

(_____)$x$ = _____ Divide both sides by _____.

$$\frac{\square}{\square} = \frac{\square}{\square}$$

$x$ = _____

Check your answer by seeing if the value you found for $x$ makes the original equation true.

(_____)$x$ + 50 = _____

_____(_____) + 50 $\stackrel{?}{=}$ _____

_____ + _____ $\stackrel{?}{=}$ _____

_____ = _____

Is your answer correct? _____

**The school ordered _____ cubic yards of soil.**

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## 3 LESSON PRACTICE

1. Describe the steps you would take to solve the equation $10.4x - 12.5 = -3.8$. Do not solve the equation.

2. Damon plans to buy a snowboard that costs \$160. Damon has \$40. He will save \$20 each week until he has enough to buy the snowboard. Let $x$ represent the number of weeks it will take for Damon to save the money he needs.

   Circle the numbers to complete the equation that models this situation. Then circle the solution.

| | | | | | | |
|---|---|---|---|---|---|---|
| $20x$ | | $20$ | | $60$ | | $2$ |
| $40x$ | $+$ | $40$ | $=$ | $160$ | $x =$ | $3.5$ |
| $160x$ | | $160$ | | $200$ | | $4.5$ |
| | | | | | | $6$ |

3. Lydia's gas tank was $\frac{3}{4}$ full when she left on a trip. She used 8 gallons of gasoline during her trip. Now, 5.5 gallons of gasoline remain in the gas tank. How many gallons of gasoline does the tank of Lydia's car hold? Show your work.

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4. Hayden's checking account balance was $145.95. After two checks were cashed, each for the same amount, his balance was −$4.05. This problem can be represented by the equation below:

$$145.95 - 2x = -4.05$$

Solve the equation for $x$. Show your work. Then explain what $x$ represents in the problem.

5. The perimeter of a rectangle is 44 meters. Its width is 10 meters.

| Angelo's Solution | Fae's Solution |
|---|---|
| I knew that the perimeter is equal to twice the length plus twice the width. So, I worked backwards and subtracted twice the width from the perimeter.<br>$44 - (2 \times 10) = 44 - 20 = 24$<br>I knew that 24 was the sum of the two longer sides, so I divided by 2:<br>$24 \div 2 = 12$<br>The length is 12 meters. | $P = 2l + 2w$<br>$44 = 2l + 2(10)$<br>$44 = 2l + 20$<br>$44 - 20 = 2l + 20 - 20$<br>$24 = 2l$<br>$\frac{24}{2} = \frac{2l}{2}$<br>$12 = l$ |

Both Angelo and Fae got the same value for the length. Are both of their methods correct? How are the two methods similar? How are they different?

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**6** Use numbers from the box to write equations that have the given solutions. You can use a number more than once.

| 1 |
|---|
| 3 |
| 4 |
| 6 |
| 12 |

Equation 1: $1.5x +$ _____ $= 12$ Solution: $x = 4$

Equation 2: $4(x -$ _____$) = 16$ Solution: $x = 7$

**7** Li uses $1\frac{1}{2}$ yards of fabric to make a small bag and $2\frac{1}{4}$ yards of fabric to make a large bag. Li has $14\frac{1}{4}$ yards of fabric. She decides to make one large bag and use the rest of the fabric for small bags. Write and solve an algebraic equation to find the number of small bags Li can make. Show your work.

**9** Consider the two equations.

Equation A: $1.4(x + 3) = 9$ Equation B: $1.4x + 3 = 9$

How are the two equations similar? How are they different? How do the differences between these two equations affect how you solve for $x$? Explain.

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10 Adrian is visiting a friend in Canada. Today's temperature is expected to reach 13°C. Adrian uses the equation $13 = \frac{5}{9}(x - 32)$ to convert this temperature to degrees Fahrenheit. Let $x$ represent the temperature in degrees Fahrenheit. What is the temperature in degrees Fahrenheit? Show your work.

11 Marco has a gift card to a sporting goods store valued at $90. He buys a sweatshirt for $20 and several pairs of running shorts for $16 each. He uses a coupon for 10% off the entire purchase. He uses the entire value of his gift card.

**Part A**

Marco says that a coupon for 10% off means paying 90% of the total amount. He wrote this equation to represent $p$, the number of pairs of running shorts he bought.

$$0.9(90) = 20 + 16p$$

Marco's equation is incorrect and could not be used to determine the price of one pair of running shorts. Explain why.

**Part B**

Write and solve an equation that will find the correct value for $p$. Show your work.

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